

1. Write down an equation to show that the circumference of a circle is proportional to its radius. What is the constant of proportionality?

$$C = k \cdot r \text{ where the constant of proportionality } k \text{ is equal to } 2\pi$$

2. Write down an equation to show that the area of an equilateral triangle is proportional to the square of its side length. What is the constant of proportionality?

$$A = k \cdot s^2 \text{ where the constant of proportionality } k \text{ is equal to } \sqrt{3}/4$$

3. After the brakes are applied in an automobile, it will still travel a certain distance before coming to rest. This is referred to as the automobile's *stopping distance*, and it is directly proportional to the square of the automobile's speed. If an automobile has a stopping distance of 45 feet when traveling at 30 miles per hour, then what is the stopping distance of the same automobile traveling at 60 miles per hour?

180 feet

4. The population of a town is currently 4000. Letting  $P$  represent the town's population  $t$  years from now, write down a differential equation with initial value to model the population under the following conditions.

- (a) The population is growing at a rate of 40 people per year.

$$\frac{dP}{dt} = 40 \quad \text{and} \quad P(0) = 4000$$

- (b) The population is growing at a rate which is proportional to the population size with a constant of proportionality of 0.05.

$$\frac{dP}{dt} = 0.05P \quad \text{and} \quad P(0) = 4000$$

5. Suppose that 500 northern pike are released into a man-made lake which had no northern pike beforehand. Write down a differential equation with initial value to model the number of these fish under the following conditions.

- (a) This fish population decreases by 40 fish per year.

$$\frac{dP}{dt} = -40 \quad \text{and} \quad P(0) = 500$$

- (b) This fish population grows at a continuous growth rate of 6% per year.

$$\frac{dP}{dt} = 0.06P \quad \text{and} \quad P(0) = 500$$

6. Alice was standing in a room with a 12 foot ceiling. She is normally only 4 feet tall, but after drinking liquid from a strange bottle, she started to grow at a rate which is proportional to the product of her height and the distance from the top of her head to the ceiling. If  $h$  represents Alice's height at time  $t$ , then find the differential equation which models her height.

$$\frac{dh}{dt} = kh(12 - h) \quad \text{and} \quad h(0) = 4$$

7. Newton's Law of Cooling states that the rate at which an object cools is proportional to the difference between its temperature and that of its surroundings.

- (a) Using  $T$  for temperature at time  $t$ ,  $k$  for the constant of proportionality, and  $T_s$  for the surrounding temperature, determine a differential equation which models the object's temperature.

see worksheet B

- (b) A fresh cup of coffee has a temperature of  $90^\circ C$  and is brought into a room where the temperature is  $20^\circ C$ . Suppose  $k$  has the value of  $-0.1^\circ C$  per minute per  $^\circ C$  of temperature difference. Use Euler's Method to approximate the coffee's temperature in 10 minutes.

see worksheet B