

1. Given the following initial value problem, use Euler's Method with $\Delta t = 2$ to estimate $w(6)$.

$$\frac{dw}{dt} = \ln(w + 1), \quad w(0) = 10$$

t_{old}	w_{old}	w'_{old}	$w_{new} \approx w_{old} + w'_{old} \cdot \Delta t$
0	10	2.4	14.8
2	14.8	2.8	20.3
4	20.3	3.1	26.4
6	26.4		

Using Euler's Method with $\Delta t = 2$, we obtain the estimate $w(6) \approx 26.4$

2. Suppose y is a function of t which satisfies the differential equation

$$\frac{dy}{dt} = \frac{4(y - 5)(y - 20)}{21}$$

- (a) Sketch a rough graph of y given that $y(0) = 15$

graph not shown

- (b) Find all real values of y for which the quantity y is increasing.

$$y < 5 \text{ or } y > 20$$

- (c) Find all real values of y for which the quantity y is decreasing.

$$5 < y < 20$$

- (d) For which values of y is the quantity y in equilibrium? Determine whether each of these equilibrium values is stable or unstable.

$y = 5$ is a stable equilibrium point and $y = 20$ is an unstable equilibrium point

3. Using P for your dependent variable, t for your independent variable, and k , r , m , or C for any necessary constants, write down the general form for a differential equation which models each of the following types of growth.

(a) logistic growth (see worksheet C)

(b) linear growth (see worksheet C)

(c) exponential growth (see worksheet C)

4. Suppose that 500 trout are released into a man-made lake which had no trout before. Further suppose that the trout population, P , grows logistically according to the following differential equation where t represents the number of years since the initial release of the trout.

$$\frac{dP}{dt} = 0.1P \left(1 - \frac{P}{2500} \right) \quad \text{and} \quad P(0) = 500$$

(a) As a percentage, what is the intrinsic growth rate of this trout population?

10%

(b) What is the carrying capacity for this trout population?

2500 trout

(c) Sketch a rough graph of this trout population being sure to show any long-term behavior.

graph not shown