Math 172 **Quiz 10 Solutions** **Spring 2006**

1. (a) 0.2 (i.e., 20%)
   (b) No
   (c) $R = 0.2$, so $u(n) = u(n-1) + 0.2 \cdot u(n-1)$
   (d) $E = 0$ (unstable)
   (e) graph not shown
   (f) N/A
   (g) N/A
   (h) No
2. (a) 0.2 (i.e., 20%)
   (b) 800
   (c) $R = \frac{-0.2}{800}u + 0.2$, so $u(n) = u(n-1) + \left(\frac{-0.2}{800}u(n-1) + 0.2\right) \cdot u(n-1)$
   (d) $E = 0$ (unstable), $E = 800$ (stable)
   (e) graph not shown
   (f) For $E = 800$, the maximum interval of stability is $(0, 4800)$
   (g) For $E = 800$ and an appropriate choice for $u(0)$ we obtain $\lim_{n\to\infty} \frac{u(n) - E}{u(n-1) - E} = 0.8$. Thus for large enough n, $u(n)$ is about 20% closer to equilibrium than $u(n-1)$.
   (h) No
3. (a) 0.2 (i.e., 20%)
   (b) 800
   (c) $R = \frac{-0.2}{800^2}u^2 + 0.2$, so $u(n) = u(n-1) + \left(\frac{-0.2}{800^2}u^2(n-1) + 0.2\right) \cdot u(n-1)$
   (d) Disregarding $E = -800$ we get $E = 0$ (unstable), $E = 800$ (stable)
   (e) graph not shown
   (f) For $E = 800$, the maximum interval of stability is $\left(0, 800\sqrt{6}\right) \approx (0, 1959.59)$
   (g) For $E = 800$ and an appropriate choice for $u(0)$ we obtain $\lim_{n\to\infty} \frac{u(n) - E}{u(n-1) - E} = 0.6$ so for large enough n, $u(n)$ is about 40% closer to equilibrium than $u(n-1)$.
   (h) No
4. (a) 0.2 (i.e., 20%)
   (b) 800
   (c) $R = \frac{-0.2}{500^2}(u - 300)^2 + 0.2$, so $u(n) = u(n-1) + \left(\frac{-0.2}{500^2}(u(n-1) - 300)^2 + 0.2\right) \cdot u(n-1)$
   (d) Disregarding $E = -200$ we get $E = 0$ (unstable), $E = 800$ (stable)
   (e) graph not shown

(f) For $E = 800$, the maximum interval of stability is $\left(0, 300 + 500\sqrt{6}\right) \approx (0, 1524.74)$

(g) For $E = 800$ and an appropriate choice for $u(0)$ we obtain $\lim_{n\to\infty} \dfrac{u(n) - E}{u(n-1) - E} = 0.36$ so for large enough n, $u(n)$ is about 64% closer to equilibrium than $u(n-1)$.

(h) No

5. (a) 0.2 (i.e., 20%)

(b) 800

(c) $R = \frac{-0.2}{300^2}(u - 500)^2 + 0.2$, so $u(n) = u(n-1) + \left(\frac{-0.2}{300^2}(u(n-1) - 500)^2 + 0.2\right) \cdot u(n-1)$

(d) $E = 0$ (stable), $E = 200$ (unstable), $E = 800$ (stable)

(e) graph not shown

(f) For $E = 800$, the maximum interval of stability is $\left(200, 400 + 100\sqrt{61}\right) \approx (200, 1181.02)$

(g) For $E = 800$ and an appropriate choice for $u(0)$ we obtain $\lim_{n\to\infty} \dfrac{u(n) - E}{u(n-1) - E} = -0.067$ so for large enough n, $u(n)$ oscillates around its equilibrium value and is about 93.3% closer to equilibrium than $u(n-1)$.

(h) Yes, 200 is the minimum viable population.