

2. The formula for the parabola is  $R = -\frac{0.7}{200^2}u^2 + 0.7$ , so the discrete dynamical system is

$$u(n) = u(n-1) + \left(-\frac{0.7}{200^2}u^2(n-1) + 0.7\right) \cdot u(n-1)$$

The equilibrium values are  $-2000$ ,  $0$ , and  $2000$ . You should be able to quickly find these without doing any algebra. The equilibrium value  $2000$  is stable and its maximum interval of stability is  $(0, 2000\sqrt{17/7}) \approx (0, 3116.77)$ . If we let  $u(0)$  be a number in this interval other than  $2000$ , we find that

$$\lim_{n \rightarrow \infty} \frac{u(n) - 2000}{u(n-1) - 2000} = -0.4$$

So for large enough  $n$ ,  $u(n)$  oscillates about its equilibrium value of  $2000$  and gets approximately 60% closer each time period.

6. (a)  $a = -\frac{0.2}{100^4}$ ,  $r = 0.2 - \frac{0.2}{100^4}u^4$ , graph not shown.
- (b)  $u(n) = u(n-1) + \left(0.2 - \frac{0.2}{100^4}u^4(n-1)\right) \cdot u(n-1)$  has equilibrium values  $-100$ ,  $0$ , and  $100$ . You should be able to quickly find these without doing any algebra.
- (c) The equilibrium value  $0$  is unstable. The equilibrium value  $100$  is stable and its maximum interval of stability is  $(0, 100\sqrt[4]{6}) \approx (0, 156.5)$ . If we let  $u(0)$  be a number in this interval other than  $100$ , we find that

$$\lim_{n \rightarrow \infty} \frac{u(n) - 100}{u(n-1) - 100} = 0.2$$

So for large enough  $n$ ,  $u(n)$  approaches its equilibrium value of  $100$  and gets approximately 80% closer each time period.

10. (a)  $r = -\frac{0.3}{1000^2}(u-1200)^2 + 0.3$ . On the graph of this parabola one sees that the growth rate is negative for  $u < 200$  or  $u > 2200$ .
- (b)  $u(n) = u(n-1) + \left(-\frac{0.3}{1000^2}(u(n-1)-1200)^2 + 0.3\right) \cdot u(n-1)$  has equilibrium values  $0$ ,  $200$ , and  $2200$ . You should be able to quickly find these without doing any algebra. The equilibrium value  $0$  is stable. The equilibrium value  $200$  is unstable. The equilibrium value  $2200$  is stable and its maximum interval of stability is  $(200, 1100 + \frac{100}{3}\sqrt{4089}) \approx (200, 3231.5)$ . If we let  $u(0)$  be a number in this interval other than  $2200$ , we find that

$$\lim_{n \rightarrow \infty} \frac{u(n) - 2200}{u(n-1) - 2200} \approx -0.32$$

So for large enough  $n$ ,  $u(n)$  oscillates about its equilibrium value of  $2200$  and gets approximately 68% closer each time period.

The minimum viable population is  $200$ .