

- LINEAR GROWTH ( $m = \text{slope}$ ,  $P_0 = \text{initial value}$ )

Type	Linear Model	Explicit Solution
continuous	$\frac{dP}{dt} = m$	$P = mt + P_0$
discrete	$P(n) = P(n-1) + m$	$P = mt + P_0$

- EXPONENTIAL GROWTH ( $r = \text{growth rate}$ ,  $P_0 = \text{initial value}$ )

Type	Exponential Model	Explicit Solution
continuous	$\frac{dP}{dt} = rP$	$P = P_0 e^{rt}$
discrete	$P(n) = P(n-1) + rP(n-1)$	$P = P_0 (1+r)^n$

- LOGISTIC GROWTH ( $r = \text{growth rate}$ ,  $k = \text{carrying capacity}$ ,  $P_0 = \text{initial value}$ )

Type	Logistic Model
continuous	$\frac{dP}{dt} = rP \left(1 - \frac{P}{k}\right)$
discrete	$P(n) = P(n-1) + rP(n-1) \left(1 - \frac{P(n-1)}{k}\right)$

1. If  $P$  represents some population  $t$  years from now, and  $\frac{dP}{dt} = 20$ , then which of the following statements is correct?
  - (a)  $P$  grows linearly by 20 people per year.
  - (b)  $P$  grows linearly by 120 people per year.
  - (c)  $P$  grows exponentially by 20% per year.
  - (d)  $P$  grows logistically with a carrying capacity of 20.
2. If  $P$  represents some population  $n$  years from now, and  $P(n) = 1.2P(n - 1)$ , then which of the following statements is correct?
  - (a)  $P$  grows linearly by 20 people per year.
  - (b)  $P$  grows linearly by 120 people per year.
  - (c)  $P$  grows exponentially by 20% per year.
  - (d)  $P$  grows logistically with a carrying capacity of 120.
3. The population of a city was 5000 in 1980. Since then the population has been increasing by 100 people per year.
  - (a) Determine a discrete dynamical system with initial value to model the city's population.
  - (b) Determine a differential equation with initial value to model the city's population.
  - (c) Determine an explicit formula for the city's population.
  - (d) What does your model predict for the city's population in the year 2000?
  - (e) When does your model predict the population will have reached 10000?
4. An initial deposit of \$200 is made into an account with an annual percentage rate (APR) of 3% compounded annually.
  - (a) Determine a discrete dynamical system with initial value to model the amount of money in this account.
  - (b) Determine an explicit formula for the amount of money in this account.
  - (c) How much money will the account hold 8 years after the initial deposit?
  - (d) How long will it take until the balance in this account is \$500?
5. An initial deposit of 200 is made into an account with an annual percentage rate (APR) of 3% compounded continuously.
  - (a) Determine a differential equation with initial value to model the amount of money in this account.
  - (b) Determine an explicit formula for the amount of money in this account.
  - (c) How much money will the account hold 8 years after the initial deposit?
  - (d) How long will it take until the balance in this account is \$500?
6. Why did I suggest using a discrete dynamical system for problem (4), but a differential equation for problem (5)?

7. The number of fish in a pond was 300 in 1970. Since then the number of fish has been increasing by 5% per year.
  - (a) Determine a discrete dynamical system with initial value to model the number of fish in this pond.
  - (b) Enter this system into your calculator to make a table of values for the number of fish in the pond each year from 1970 to 1980.
  - (c) Determine an explicit formula for the number of fish in the pond.
  - (d) When will the fish population reach 750?
8. The number of fish in a pond was 300 in 1970. Since then the number of fish has been increasing by 5% per year.
  - (a) Determine a differential equation with initial value to model the number of fish in this pond.
  - (b) Use Euler's Method with  $\Delta t = 1$  to make a table of values for the number of fish in the pond each year from 1970 to 1980.
  - (c) Determine an explicit formula for the number of fish in the pond.
  - (d) When will the fish population reach 750?
9. Which model was best to use for the fish population – the discrete dynamical system or the differential equation? Why?
10. There are currently 5000 deer in a forest. Suppose the population of deer grows logistically with an intrinsic growth rate of 6% and a carrying capacity of 20,000.
  - (a) Sketch a rough graph of the deer population.
  - (b) Determine a discrete dynamical system with initial value to model the deer population.
  - (c) Make a table of values for the number of deer your discrete model predicts for the next 4 years.
  - (d) Determine a differential equation with initial value to model the deer population.
  - (e) Use Euler's Method with  $\Delta t = 1$  to make a table of values for the number of deer your continuous model predicts for the next 4 years.
  - (f) Use Euler's Method with  $\Delta t = 0.5$  to make a table of values for the number of deer your continuous model predicts for the next 4 years.
  - (g) How close are the approximations for discrete and continuous models?