1. Suppose y is a function of x which satisfies the following differential equation.

$$\frac{dy}{dx} = 2x, \qquad y(0) = 1$$

(a) Use Euler's Method with $\Delta x = 1$ to approximate $y(2)$.

x_{old}	y_{old}	y'_{old}	$y_{new} \approx y_{old} + y'_{old} \cdot \Delta x$
0.0	1	0	1
1.0	1	2	3
2.0	3		

Using Euler's Method with $\Delta x = 1$, we obtain the estimate $y(2) \approx 3$

(b) Use Euler's Method with $\Delta x = 0.5$ to approximate $y(2)$.

x_{old}	y_{old}	y'_{old}	$y_{new} \approx y_{old} + y'_{old} \cdot \Delta x$
0.0	1	0	1
0.5	1	1	1.5
1.0	1.5	2	2.5
1.5	2.5	3	4
2.0	4		

Using Euler's Method with $\Delta x = 0.5$, we obtain the estimate $y(2) \approx 4$

(c) Use Euler's Method with $\Delta x = 0.1$ to approximate $y(2)$.

x_{old}	y_{old}	y'_{old}	$y_{new} \approx y_{old} + y'_{old} \cdot \Delta x$
0.0	1.00	0.0	1.00
0.1	1.00	0.2	1.02
0.2	1.02	0.4	1.06
0.3	1.06	0.6	1.12
0.4	1.12	0.8	1.20
0.5	1.20	1.0	1.30
0.6	1.30	1.2	1.42
0.7	1.42	1.4	1.56
0.8	1.56	1.6	1.72
0.9	1.72	1.8	1.90
1.0	1.90	2.0	2.10
1.1	2.10	2.2	2.32
1.2	2.32	2.4	2.56
1.3	2.56	2.6	2.82
1.4	2.82	2.8	3.10
1.5	3.10	3.0	3.40
1.6	3.40	3.2	3.72
1.7	3.72	3.4	4.06
1.8	4.06	3.6	4.42
1.9	4.42	3.8	4.80
2.0	4.80		

Using Euler's Method with $\Delta x = 0.1$, we obtain the estimate $y(2) \approx 4.8$

(d) Can you find an explicit formula for y which satisfies the differential equation? If so, then use the formula to find the exact value of $y(2)$. How do your approximations in parts (a) – (d) compare?

With the explicit formula $y = x^2 + 1$ we obtain the exact value $y(2) = 5$

2. Suppose P is a function of t which satisfies the following differential equation.

$$\frac{dP}{dt} = 0.1P, \qquad P(0) = 100$$

(a) Make tables similar to those used in problem #1 to approximate $P(3)$ using Euler's Method.

t_{old}	P_{old}	P'_{old}	$P_{new} \approx P_{old} + P'_{old} \cdot \Delta t$
0.0	100	10.0	110.0
1.0	110.0	11.0	121.0
2.0	121.0	12.1	133.1
3.0	133.1		

Using Euler's Method with $\Delta t = 1$, we obtain the estimate $P(3) \approx 133.1$

t_{old}	P_{old}	P'_{old}	$P_{new} \approx P_{old} + P'_{old} \cdot \Delta t$
0.0	100	10.00	105.00
0.5	105.00	10.50	110.25
1.0	110.25	11.03	115.76
1.5	115.76	11.58	121.56
2.0	121.56	12.16	127.63
2.5	127.63	12.76	134.01
3.0	134.01		

Using Euler's Method with $\Delta t = 0.5$, we obtain the estimate $P(3) \approx 134.01$

(b) Can you find an explicit formula for P which satisfies the differential equation? If so, then use the formula to find the exact value of $P(3)$. How do your approximations compare to exact value?

With the explicit formula $P = 100e^{0.1t}$ we obtain the exact value $P(3) = 100e^{0.3} \approx 134.99$

3. This is to get you started on question #7b from **Worksheet A**. We saw in class that the appropriate differential equation is

$$\frac{dT}{dt} = -0.1(T - 20), \qquad T(0) = 90$$

You are asked to use Euler's method in order to approximate $T(10)$. You are not given a value for Δt, but for each value that you choose, you should make a table similar to those used in our last problem.

(a) In the following table, what is the value of Δt ? Fill in appropriate column headings using correct variable names. Now complete the table in order to approximate $T(10)$.

t_{old}	T_{old}	T'_{old}	$T_{new} \approx T_{old} + T'_{old} \cdot \Delta t$
0.0	90	-7	72.5
2.5	72.5	-5.25	59.38
5.0	59.38	-3.94	49.53
7.5	49.53	-2.95	42.15
10.0	42.15		

Using Euler's Method with $\Delta t = 2.5$, we obtain the estimate $T(10) \approx 42.15$

(b) Be sure to make a couple of additional tables with smaller values chosen for Δt. If you are proficient at computer programming or using spreadsheets such as Excel, then your smallest value for Δt may be as small as 0.01. The rest of us should at least be willing to use $\Delta t = 0.5$.

t_{old}	T_{old}	T'_{old}	$T_{new} \approx T_{old} + T'_{old} \cdot \Delta t$
0	90.00	-7.00	83.00
1	83.00	-6.30	76.70
2	76.70	-5.67	71.03
3	71.03	-5.10	65.93
4	65.93	-4.59	61.33
5	61.33	-4.13	57.20
6	57.20	-3.72	53.48
7	53.48	-3.35	50.13
8	50.13	-3.01	47.12
9	47.12	-2.71	44.41
10	44.41		

Using Euler's Method with $\Delta t = 1$, we obtain the estimate $T(10) \approx 44.41$

t_{old}	T_{old}	T'_{old}	$T_{new} \approx T_{old} + T'_{old} \cdot \Delta t$
0.0	90.00	-7.00	86.50
0.5	86.50	-6.65	83.18
1.0	83.18	-6.32	80.02
1.5	80.02	-6.00	77.02
2.0	77.02	-5.70	74.16
2.5	74.16	-5.42	71.46
3.0	71.46	-5.15	68.88
3.5	68.88	-4.89	66.44
4.0	66.44	-4.64	64.12
4.5	64.12	-4.41	61.91
5.0	61.91	-4.19	59.82
5.5	59.82	-3.98	57.83
6.0	57.83	-3.78	55.93
6.5	55.93	-3.59	54.14
7.0	54.14	-3.41	52.43
7.5	52.43	-3.24	50.81
8.0	50.81	-3.08	49.27
8.5	49.27	-2.93	47.81
9.0	47.81	-2.78	46.41
9.5	46.41	-2.64	45.09
10.0	45.09		

Using Euler's Method with $\Delta t = 0.5$, we obtain the estimate $T(10) \approx 45.09$

Although it wasn't asked for in this problem, separation of variables does lead to the explicit formula $T = 20 + 70e^{-0.1t}$. Thus we obtain the exact value $T(10) = 20 + 70e^{-1} \approx 45.75$