1. Write down an equation to show that the circumference of a circle is proportional to its radius. What is the constant of proportionality?

   $C = k \cdot r$ where the constant of proportionality k is equal to 2π

2. After the brakes are applied in an automobile, it will still travel a certain distance before coming to rest. This is referred to as the automobile's *stopping distance*, and it is directly proportional to the square of the automobile's speed. If an automobile has a stopping distance of 45 feet when traveling at 30 miles per hour, then what is the stopping distance of the same automobile traveling at 60 miles per hour?

   180 feet

3. Alice was standing in a room with a 12 foot ceiling. She is normally only 4 feet tall, but after drinking liquid from a strange bottle, she started to grow at a rate which is proportional to the product of her height and the distance from the top of her head to the ceiling. If h represents Alice's height at time t, then find the differential equation which models her height.

   $\frac{dh}{dt} = kh(12 - h)$ and $h(0) = 4$

4. Consider the population model

$$\frac{dP}{dt} = 0.3P\left(1 - \frac{P}{200}\right)\left(\frac{P}{50} - 1\right)$$

   where P is the population at time t.

   (a) For which values of P is the population increasing?

   $50 < P < 200$ (also true mathematically for $P < 0$ but this won't occur for our population model)

   (b) For which values of P is the population decreasing?

   $0 < P < 50$ or $P > 200$

   (c) For which values of P is the population in equilibrium? Determine whether each of these equilibrium values is stable or unstable.

   We have an unstable equilibrium value at $P = 50$ and stable equilibria values at $P = 0$ and $P = 200$

   (d) Sketch a rough graph of P given that: (i) $P(0) = 10$, (ii) $P(0) = 50$, (iii) $P(0) = 150$, (iv) $P(0) = 300$.

   graph not shown here

5. What is the differential equation which models (i) linear growth? (ii) exponential growth? (iii) logistic growth?

   see worksheet C

6. Find the equation of the line tangent to the graph of $y = \ln(x)$ at $x = 1$. Use this to approximate the values of $\ln(0.9)$, $\ln(1.1)$ and $\ln(1.5)$ without using a calculator. Compare your approximations to the values given by your calculator for each of these values. Which of your approximations is closest to the true value? Why?

not covered this semester

7. Newton's Law of Cooling states that the rate at which an object cools is proportional to the difference between its temperature and that of its surroundings.

   (a) Using T for temperature at time t, k for the constant of proportionality, and T_s for the surrounding temperature, determine a differential equation which models the object's temperature.

   see worksheet B

   (b) A fresh cup of coffee has a temperature of $90°C$ and is brought into a room where the temperature is $20°C$. Suppose k has the value of $-0.1°C$ per minute per $°C$ of temperature difference. Use Euler's Method to approximate the coffee's temperature in 10 minutes.

   see worksheet B