1. Suppose you borrow $\$120,000$ at an 8.4% annual interest rate compounded monthly to be paid back in monthly payments of \$1800.

   (a) Write down a discrete dynamical system with initial condition to represent the balance of the loan just after each month's payment.

   *Solution:*

   Let $u(n)$ represent the amount owed $n$ years after obtaining the loan. Then

   $u(n) = u(n-1) + (0.084/12)u(n-1) - 1800 \quad \text{and} \quad u(0) = 120,000$

   or in simplified form

   $\boxed{\text{u(n)=1.007u(n-1) - 1800 and u(0) = 120,000}}$

   (b) How many months will it take to pay back the loan?

   *Solution:*

   We find that $u(90) = 206.51 \quad \text{and} \quad u(91) = -1592.04$

   It will take $\boxed{\text{91 months}}$, but if each payment is \$1800, then we have overpaid the loan by the \$1592.04.

   (c) The last payment will be a bit different than each of the preceding monthly payments. What will be the amount of this last payment?

   *Solution:*

   The first 90 payments will be for \$1800 each, but the last (91st) payment will be for $\$1800 - \$1592.04 = \boxed{\$207.96}$.

2. Find the equilibrium value for the following dynamical system.

$$u(n) = 0.9u(n-1) - 3.5$$

   *Solution:*

   Solving the equation $E = 0.9E - 3.5$, we get $E = -35$, so the equilibrium value for $u$ is $-35$.

3. Find the equilibrium value for the following dynamical system.

$$u(n) = 1.25u(n-1) - 6.1$$

   *Solution:*

   Solving the equation $E = 1.25E - 6.1$, we get $E = 24.4$, so the equilibrium value for $u$ is 24.4.

4. Find the equilibrium point for the following dynamical system.

$$u(n) = 2u(n-1) + v(n-1) + 3$$
$$v(n) = 4u(n-1) - v(n-1) + 6$$

*Solution:*

Solving the equations $E = 2E + F + 3$ and $F = 4E - F + 6$, we get $E = -2$ and $F = -1$, so the equilibrium point for $(u, v)$ is $(-2, -1)$.