1. Given the dynamical system $p(n) = 0.8p(n-1) + 16$ with $p(0) = 30$, find values for $p(1)$, $p(2)$, $p(3)$, and $p(100)$.

   Solution:

   $p(1) = 0.8 \cdot p(0) + 16 = 0.8 \cdot 30 + 16 = 40$
   $p(2) = 0.8 \cdot p(1) + 16 = 0.8 \cdot 40 + 16 = 48$
   $p(3) = 0.8 \cdot p(2) + 16 = 0.8 \cdot 48 + 16 = 54.4$

So p(1)=40, p(2)=48, and p(3)=54.4

   To avoid lengthy calculations, we may also obtain these results by entering the system into the calculator. We then obtain $p(100) \approx 80$.

2. Consider the following dynamical system of two equations.

   $$u(n) = 0.6u(n-1) + 0.3v(n-1) + 5$$
   $$v(n) = 0.5u(n-1) + 0.6v(n-1) + 3$$

   If $u(0) = 20$ and $v(0) = 15$, then determine $u(3)$ and $v(3)$. Also determine $u(70)$ and $v(70)$.

   Solution:

   $u(1) = 0.6 \cdot u(0) + 0.3 \cdot v(0) + 5 = 0.6 \cdot 20 + 0.3 \cdot 15 + 5 = 21.5$
   $v(1) = 0.5 \cdot u(0) + 0.6 \cdot v(0) + 3 = 0.5 \cdot 20 + 0.6 \cdot 15 + 3 = 22$

   $u(2) = 0.6 \cdot u(1) + 0.3 \cdot v(1) + 5 = 0.6 \cdot 21.5 + 0.3 \cdot 22 + 5 = 24.5$
   $v(2) = 0.5 \cdot u(1) + 0.6 \cdot v(1) + 3 = 0.5 \cdot 21.5 + 0.6 \cdot 22 + 3 = 26.95$

   $u(3) = 0.6 \cdot u(2) + 0.3 \cdot v(2) + 5 = 0.6 \cdot 24.5 + 0.3 \cdot 26.95 + 5 = 27.785$
   $v(3) = 0.5 \cdot u(2) + 0.6 \cdot v(2) + 3 = 0.5 \cdot 24.5 + 0.6 \cdot 26.95 + 3 = 31.42$

So u(3) = 27.785 and v(3) = 31.42

   To avoid lengthy calculations, we may also obtain these results by entering the system into the calculator. We then obtain $u(70) \approx 178.64$ and $v(70) \approx 226.23$.

3. Let $h(n)$ represent the height of a stack of n chairs. Each chair by itself is 3 feet high, but when stacked, the height of an existing stack only increases by 8 inches for each additional chair. A pattern doesn't really begin until you actually have one chair, so we won't define $h(0)$ but will start with $h(1) = 3$. Be consistent with your units and do the following:

(a) Develop a discrete dynamical system for $h(n)$.

Solution 1 (feet): Let $h(n)$ represent the height in feet of n stacked chairs with $n \geq 1$. Then

h(n) = h(n-1) + 2/3 and h(1) = 3

Solution 2 (inches): Let $h(n)$ represent the height in inches of n stacked chairs with $n \geq 1$. Then h(n) = h(n-1) + 8 and h(1) = 36

(b) Find an explicit formula for $h(n)$.

Solution 1 (feet): $h(n) = \frac{2}{3}(n-1) + 3$ or $h(n) = \frac{2}{3}n + \frac{7}{3}$

Solution 2 (inches): $h(n) = 8(n-1) + 36$ or $h(n) = 8n + 28$

4. Develop a discrete model in which 80% of some drug in the bloodstream from one day to the next is used up, but the remainder is reinforced with a maintenance dose of 40 mg per day. The initial dose is 10 mg.

Solution:

Let $u(n)$ represent the number of milligrams of drug in the bloodstream n days after the initial dose. Then

u(n)=u(n-1) - 0.8u(n-1) + 40 and u(0) = 10

or in simplified form

u(n)=0.2u(n-1) + 40 and u(0) = 10

5. There are 2 drugs, A and B. Let $a(n)$ and $b(n)$ represent the number of milligrams of each drug in the body at the beginning of day n. The body converts 10% of A into B each day and converts 25% of B into A each day. Assume that 800 mg of A and 600 mg of B are consumed each day, and that the body eliminates 100 mg of A and 200 mg of B each day. Begin with $a(0) = 800$ and $b(0) = 600$ and develop a discrete model to represent $a(n)$ and $b(n)$.

Solution:

a(n)=a(n-1) - 0.1a(n-1) + 0.25b(n-1) + 800 - 100

b(n)=b(n-1) + 0.1a(n-1) - 0.25b(n-1) + 600 - 200

a(0)=800 and b(0)=600

or in simplified form

a(n)=0.9a(n-1) + 0.25b(n-1) + 700

b(n)=0.1a(n-1) + 0.75b(n-1) + 400

a(0)=800 and b(0)=600