Math 172 Practice Problems

1. Given the discrete dynamical system below with $u(0) = 6$ and $v(0) = 4$, find the values of both $u(2)$ and $v(2)$.

$$u(n) = u(n-1) + 0.5v(n-1) - 1$$
$$v(n) = 0.25u(n-1) + 1.5v(n-1) - 3$$

2. Suppose $p(n)$ represents some population n years from now, and that this population is modeled by the following discrete dynamical system.

$$p(n) = p(n-1) + 35$$
$$p(0) = 30$$

Which one of the following statements follows from this model?

(a) The population will increase by 35 people per year.
(b) The population will increase by 30 people per year.
(c) The population will increase by 35% per year.
(d) The population will increase by 30% per year.

3. Suppose $a(n)$ represents the number of milligrams of some drug in the bloodstream n hours from now, and that the amount of this drug in the bloodstream is modeled by the following discrete dynamical system.

$$a(n) = 0.3a(n-1)$$
$$a(0) = 50$$

Which one of the following statements follows from this model?

(a) The amount of drug in the bloodstream will increase by 30% per hour.
(b) The amount of drug in the bloodstream will increase by 50% per hour.
(c) The amount of drug in the bloodstream will increase by 70% per hour.
(d) The amount of drug in the bloodstream will decrease by 30% per hour.
(e) The amount of drug in the bloodstream will decrease by 50% per hour.
(f) The amount of drug in the bloodstream will decrease by 70% per hour.

4. Find the equilibrium value for the following discrete dynamical system.

$$u(n) = -0.25u(n-1) + 20$$

5. A person takes a maintenance dose of 100 mg of some drug U each day, but loses 20% of the drug each day via the kidneys. Let $u(n)$ represent the number of milligrams of U in the body at the beginning of day n. Develop a discrete dynamical system (without initial value) to represent $u(n)$.

6. There are 2 drugs, U and V. Let $u(n)$ and $v(n)$ represent the number of milligrams of each drug in the body at the beginning of day n. The body converts 5% of U into V each day and converts 20% of V into U each day. Assume that 200 mg of U and 600 mg of V are consumed each day, and that the body eliminates 100 mg of U and 50 mg of V each day. Develop a discrete dynamical system (without initial values) to represent $u(n)$ and $v(n)$.

7. Suppose a certain chemical is eliminated from the body by the kidneys and the liver. Let $u(n)$ represent the amount of this chemical in a person's bloodstream after n days. Assume that each day, the kidneys remove 25% of the chemical from the blood. Also assume that each day, the fraction of the chemical that is broken down by enzymes from the liver is given by

$$\frac{2}{5 + u(n-1)}$$

Finally, assume that each day, the person takes a dose of 200 mg of this chemical. Develop a discrete dynamical system for $u(n)$. You do not need an initial value.

8. Suppose the metabolism of some person is such that the discrete dynamical system modeling the elimination of alcohol is

$$a(n) = a(n-1) - \frac{10a(n-1)}{4 + a(n-1)} + d$$

where $a(n)$ is the amount of alcohol (in grams) in the person's bloodstream after n hours of drinking d grams of alcohol per hour.

At a 6-hour party, suppose this person drinks 28 grams of alcohol each hour for 4 hours, but then stops drinking. Fill in the remaining entries in the table below to show the amount of alchohol in the person's bloodstream at each hour.

n	a(n)
0	0
1	28.0
2	
3	
4	
5	
6	

9. Find an explicit formula for an expression which satisfies each of the following discrete dynamical systems given that $u(0) = 90$ in each case.

   (a) $u(n) = u(n-1) + 10$
   (b) $u(n) = 0.8u(n-1)$
   (c) $u(n) = 0.8u(n-1) + 10$

10. Suppose we have the following discrete dynamical system.

$$u(n) = -0.4u^2(n-1) + 3.4u(n-1) - 2$$

This system has an unstable equilibrium value at 1 and a stable equilibrium value at 5. Which of the following intervals is the maximum interval of stability for the stable equilibrium value?

   (a) $(1, 5)$
   (b) $(1, 5.5)$
   (c) $(1, 6)$
   (d) $(1, 6.5)$
   (e) $(1, 7)$
   (f) $(1, 7.5)$
   (g) $(1, 8)$
   (h) $(1, 8.5)$
   (i) $(1, 9)$
   (j) $(1, 9.5)$
   (k) $(1, 10)$

11. Suppose y is a function of t which satisfies the differential equation below.

$$\frac{dy}{dt} = \frac{6(y-3)(8-y)}{24}$$

Sketch rough graphs of y for each initial value below. Note: To save time you may draw all 4 together on one set of coordinate axes.

   (a) $y(0) = 9$
   (b) $y(0) = 8$
   (c) $y(0) = 7$
   (d) $y(0) = 0$

12. Determine a differential equation which models the growth of a city's population for each of the following possible types of growth. No initial value is needed here.

(a) The population grows by 100 people per year.

Answer: $\dfrac{dP}{dt} =$

(b) The population grows at a continuous growth rate of 5% per year.

Answer: $\dfrac{dP}{dt} =$

(c) The population grows logistically with an intrinsic growth rate of 4% per year and a carrying capacity of 8000.

Answer: $\dfrac{dP}{dt} =$

13. Find an explicit formula for y as a function of t in the following initial value problems.

(a) $\dfrac{dy}{dt} = 0.2y, \qquad y(0) = 10$

(b) $\dfrac{dy}{dt} = \dfrac{2t}{e^y}, \qquad y(0) = 0$

14. Suppose that an animal population grows logistically with an intrinsic growth rate of 20% and a carrying capacity of 300.

(a) Carefully sketch a graph of the growth rate for this population as a function of the population itself. Be sure to clearly label the values for all intercepts.

(b) Carefully sketch a graph of the population as a function of time beginning with an initial population of 50 animals. Clearly show any long term behavior.

(c) Determine a discrete dynamical system to model this population.

15. Given the following initial value problem, use Euler's Method with $\Delta t = 2$ to make estimates for P at the times indicated in the table.

$$\frac{dP}{dt} = \frac{1}{2}\sqrt{P}, \qquad P(0) = 100$$

t	P		
0.0	100		
2.0			
4.0			
6.0			
8.0			