

Name _____

Row # _____ Seat # _____

- Do not open this test booklet until told to do so.
- Turn off all cell phones.
- For multiple-choice questions, precisely one answer is correct. Circle this correct answer.
- For all other questions, you must show sufficient work to justify your answer.
- You are not allowed to borrow another student's calculator during the test.

#1 (6 points) _____

#2 (6 points) _____

#3 (6 points) _____

#4 (6 points) _____

#5 (6 points) _____

#6 (10 points) _____

#7 (8 points) _____

#8 (10 points) _____

#9 (8 points) _____

#10 (10 points) _____

#11 (10 points) _____

#12 (7 points) _____

#13 (7 points) _____

Total (100 points) _____

1. (6 points) On the graph of $y = 5x + 6 - 20 \ln(x)$, what is the slope of the curve at $x = 10$?

2. (6 points) If $y = \frac{x}{e^x}$, then

$$\frac{dy}{dx} =$$

3. (6 points) If $P(t) = e^{(t^2+5)}$, then

$$P'(t) =$$

4. (6 points) For Nancy's metabolism, the dynamical system modeling her elimination of alcohol is

$$a(n) = a(n-1) - \frac{9a(n-1)}{5 + a(n-1)} + 12$$

where $a(n)$ is the amount of alcohol (in grams) in her bloodstream after n hours of drinking 12 grams of alcohol each hour. Find the equilibrium value for this dynamical system.

5. (6 points) For Nancy's metabolism, the dynamical system modeling her elimination of alcohol is

$$a(n) = a(n-1) - \frac{9a(n-1)}{5 + a(n-1)} + 12$$

where $a(n)$ is the amount of alcohol (in grams) in her bloodstream after n hours of drinking 12 grams of alcohol each hour. Suppose that Nancy drinks 12 grams of alcohol each hour at a 4 hour party. Begin with $a(0) = 0$ and make a table of values which shows the amount of alcohol in her bloodstream at each hour from $n = 0$ to $n = 4$.

6. (10 points) Given the following initial value problem, use Euler's Method with $\Delta t = 2$ to make estimates for P at the times indicated in the table.

$$\frac{dP}{dt} = \frac{1}{2}\sqrt{P}, \quad P(0) = 100$$

t	P		
0.0	100		
2.0			
4.0			
6.0			
8.0			

7. (8 points) Find all equilibrium values for the following differential equation. There is no need to discuss whether or not these equilibrium values are stable.

$$\frac{dP}{dt} = 0.5(P - 4)(P + 3)(P^2 - 25)$$

8. (10 points) Suppose y is a function of t which satisfies the differential equation below.

$$\frac{dy}{dt} = 0.25(y - 10)(20 - y)$$

Sketch plausible graphs for y as a function of t given each initial value below. Your graphs should clearly show if the y -values approach any particular values (i.e. horizontal asymptotes). You should draw all five graphs together on one set of coordinate axes.

(a) $y(0) = 25$

(b) $y(0) = 20$

(c) $y(0) = 15$

(d) $y(0) = 10$

(e) $y(0) = 5$

9. (8 points) Suppose that 200 rabbits are released on Lady Tottington's estate, and that the population of these rabbits grows logistically with an intrinsic growth rate of 4% per month and a carrying capacity of 1000 rabbits.

(a) Let R represent the number of rabbits on her estate t months after the initial release. Determine a differential equation with initial condition to model this population of rabbits.

(b) Sketch a plausible graph for this population of rabbits, being sure to include intercepts as well as long term behaviour.

10. (10 points) The population of a town is currently 600, but is projected to grow by 30 people per year from now on.

(a) Determine a differential equation with initial condition to model this town's population. Use P for the population t years from now.

(b) Find an explicit formula for this town's population.

11. (10 points) The population of a town is currently 400, but is projected to grow at a continuous rate of 20% per year from now on.

(a) Determine a differential equation with initial condition to model this town's population. Use P for the population t years from now.

(b) Find an explicit formula for this town's population.

12. (7 points) Suppose that the population of a town is always growing at a rate which is proportional to the population itself. Suppose further that the population is currently 500 and is growing at a rate of 25 people per year. Find a differential equation with initial condition to model the population of this town. Use P for the population t years from now.

13. (7 points) Suppose a certain chemical is eliminated from the body by the kidneys and the liver. Let $u(n)$ represent the amount of this chemical in a person's bloodstream after n days. Assume that each day, the kidneys remove 20% of the chemical from the blood. Also assume that each day, the fraction of the chemical that is broken down by enzymes from the liver is given by

$$\frac{4}{9 + u(n-1)}$$

Finally, assume that each day, the person takes a dose of 180 mg of this chemical. Develop a dynamical system for $u(n)$. You do not need an initial value.