Math 172 **Modeling Alcohol Levels - Solutions** **Fall 2006**

For chemicals, such as caffeine, in which a fixed percentage is eliminated by the kidneys each time period, the amount of the chemical in the body is modeled by the dynamical system

$$a(n) = a(n-1) - r \cdot a(n-1) + d$$

where r is the elimination rate and d is the amount of the chemical consumed in each time period. For caffeine, $r \approx 0.13$.

For alcohol, the elimination rate is not a constant fixed percentage. Instead it depends upon the amount of alcohol already present in the body. With more alcohol in the body, the liver becomes less efficient in breaking down that alcohol. For men, a reasonable value for the elimination rate is $r = 10/(4 + a)$. The elimination rate is lower for women since they have less dehydrogenase, a liver enzyme that breaks down alcohol, than men.

For men we will therefore use the dynamical system

$$a(n) = a(n-1) - \frac{10}{4 + a(n-1)} \cdot a(n-1) + d$$

where $a(n)$ represents the number of grams of alcohol in the bloodstream after n hours of drinking d grams of alcohol per hour.

Using $a(0) = 0$, fill in the rest of the table below to model the number of grams of alcohol in the bloodstream over the course of a 5-hour party. To determine the value of d, note that each drink (one 12-ounce beer, 5 ounces of wine, or 1 ounce of 100-proof liquor) contains 14 grams of alcohol.

Grams of Alcohol in Bloodstream

# hours	1/2 drink per hour	1 drink per hour	2 drinks per hour	4 drinks per hour
0	0	0	0	0
1	7	14	28	56
2	7.6	20.2	47.3	102.7
3	8.1	25.9	66.0	149.0
4	8.4	31.2	84.6	195.3
5	8.6	36.3	103.1	241.5

For a 140-pound male, 40 grams of alcohol in the bloodstream gives a blood alcohol content of 0.08. This is one level used by many states to determine a DUI conviction. With 200-250 grams of alcohol in the bloodstream one risks coma or even death.