

1. Suppose y is a function of x which satisfies the following differential equation.

$$\frac{dy}{dx} = 2x, \quad y(0) = 1$$

- (a) Use Euler's Method with $\Delta x = 1$ to approximate $y(2)$.

x_{old}	y_{old}	y'_{old}	$y_{new} \approx y_{old} + y'_{old} \cdot \Delta x$
0.0	1		
1.0			
2.0			

- (b) Use Euler's Method with $\Delta x = 0.5$ to approximate $y(2)$.

x_{old}	y_{old}	y'_{old}	$y_{new} \approx y_{old} + y'_{old} \cdot \Delta x$
0.0	1		
0.5			
1.0			
1.5			
2.0			

(c) Use Euler's Method with $\Delta x = 0.2$ to approximate $y(2)$.

x_{old}	y_{old}	y'_{old}	$y_{new} \approx y_{old} + y'_{old} \cdot \Delta x$
0.0	1		
0.2			
0.4			
0.6			
0.8			
1.0			
1.2			
1.4			
1.6			
1.8			
2.0			

(d) Use Euler's Method with $\Delta x = 0.1$ to approximate $y(2)$.

x_{old}	y_{old}	y'_{old}	$y_{new} \approx y_{old} + y'_{old} \cdot \Delta x$
0.0	1		
0.1			
0.2			
0.3			
0.4			
0.5			
0.6			
0.7			
0.8			
0.9			
1.0			
1.1			
1.2			
1.3			
1.4			
1.5			
1.6			
1.7			
1.8			
1.9			
2.0			

(e) Can you find an explicit formula for y which satisfies the differential equation? If so, then use the formula to find the exact value of $y(2)$. How do your approximations in parts (a) – (d) compare?

2. This is to get you started on question #7b from **Worksheet A**. We saw in class that the appropriate differential equation is

$$\frac{dT}{dt} = -0.1(T - 20), \quad T(0) = 90$$

You are asked to use Euler's method in order to approximate $T(10)$. You are not given a value for Δt , but for each value that you choose, you should make a table similar to those used in our last problem.

- (a) In the following table, what is the value of Δt ? Fill in appropriate column headings using correct variable names. Now complete the table in order to approximate $T(10)$.

t_{old}	T_{old}		
0.0	90	-7	72.5
2.5	72.5		
5.0			
7.5			
10.0			

- (b) Be sure to make a couple of additional tables with smaller values chosen for Δt . If you are proficient at computer programming or using spreadsheets such as Excel, then your smallest value for Δt may be as small as 0.01. The rest of us should at least be willing to use $\Delta t = 0.5$.
- (c) Can you find an explicit formula for T which satisfies the differential equation? Don't worry if you are unable to do this. It is more difficult than the previous problem. If you are able to find an explicit formula, then use it to find the exact value of $T(10)$. How do your approximations compare to the exact value?