

1. Suppose y is a function of x which satisfies the following differential equation.

$$\frac{dy}{dx} = 2x, \quad y(0) = 1$$

(a) Using Euler's Method with $\Delta x = 1$ gives $y(2) \approx 3$.

x_{old}	y_{old}	y'_{old}	$y_{new} \approx y_{old} + y'_{old} \cdot \Delta x$
0.0	1	0	1
1.0	1	2	3
2.0	3		

(b) Using Euler's Method with $\Delta x = 0.5$ gives $y(2) \approx 4$.

x_{old}	y_{old}	y'_{old}	$y_{new} \approx y_{old} + y'_{old} \cdot \Delta x$
0.0	1	0	1
0.5	1	1	1.5
1.0	1.5	2	2.5
1.5	2.5	3	4
2.0	4		

(c) Using Euler's Method with $\Delta x = 0.2$ gives $y(2) \approx 4.6$.

x_{old}	y_{old}	y'_{old}	$y_{new} \approx y_{old} + y'_{old} \cdot \Delta x$
0.0	1	0.0	1
0.2	1	0.4	1.08
0.4	1.08	0.8	1.24
0.6	1.24	1.2	1.48
0.8	1.48	1.6	1.8
1.0	1.8	2.0	2.2
1.2	2.2	2.4	2.68
1.4	2.68	2.8	3.24
1.6	3.24	3.2	3.88
1.8	3.88	3.6	4.6
2.0	4.6		

(d) In class on Wednesday we'll compare our answers using $\Delta t = 0.1$.

(e) An explicit solution to this differential equation is $y = x^2 + 1$ which results in the exact value $y(2) = 5$. How close did you get in your approximation using $\Delta x = 0.1$? Did anyone go further and use $\Delta x = 0.01$?