

Name _____

1. (4 points) Suppose a certain chemical is eliminated from the body by the kidneys and the liver. Let $u(n)$ represent the amount of this chemical in a person's bloodstream after n days. Assume that each day, the kidneys remove 10% of the chemical from the blood. Also assume that each day, the fraction of the chemical that is broken down by enzymes from the liver is given by

$$\frac{2}{5 + u(n-1)}$$

Finally, assume that each day, the person takes a dose of 20 mg of this chemical. Develop a dynamical system for $u(n)$. You do not need an initial value.

2. (3 points) Suppose the metabolism of some person is such that the dynamical system modeling the elimination of alcohol is

$$a(n) = a(n - 1) - \frac{10a(n - 1)}{4 + a(n - 1)} + d$$

where $a(n)$ is the amount of alcohol (in grams) in the person's bloodstream after n hours of drinking d grams of alcohol per hour.

If this person's weight is such that 46 grams of alcohol in the bloodstream represents a blood alcohol level of 0.08 (the amount in SC for a DWI conviction), then what is the largest amount of beer this person can drink per hour at a 4-hour party if he wants to stay below that blood alcohol level of 0.08? Begin with $a(0) = 0$. You should be working with grams, but to see if your answer makes sense it may help to know that one 12-ounce can of beer contains about 14 grams of alcohol.

3. (3 points) Find the equilibrium values for the following dynamical system.

$$u(n) = -1.05 + 1.5u(n-1) - 0.05u^2(n-1)$$

Determine the stability of each equilibrium point. For one extra credit point, determine the maximum interval of stability for each stable equilibrium point.