Math 172 (Section 1) **Test 3** **November 21, 2005**

Name _____

1. (10 points) The population of an animal species is given by $P(t) = 5t^2 + 30t + 2500$ where t represents the number of months since January 1, 2004. How quickly is the population growing 1 year later?

2. (12 points) Using P for your dependent variable, t for your independent variable, and k, r, m, or C for any necessary constants, write down the general form for a differential equation which models each of the following types of growth.

   (a) logistic growth

   (b) linear growth

   (c) exponential growth

3. (5 points) If $f(x) = 3x^5 + 10$, then

$f'(x) =$

4. (5 points) If $y = \dfrac{2}{x^3}$, then

$\dfrac{dy}{dx} =$

5. (5 points) If $w = 2e^{3t}$, then

$\dfrac{dw}{dt} =$

6. (5 points) If $h = e^{-t}$, then

$\dfrac{dh}{dt} =$

7. (10 points) Suppose that 500 trout are released into a man-made lake which had no trout before. Further suppose that the trout population, P, grows logistically according to the following differential equation where t represents the number of years since the initial release of the trout.

$$\frac{dP}{dt} = 0.1P\left(1 - \frac{P}{2500}\right), \qquad P(0) = 500$$

(a) As a percentage, what is the intrinsic growth rate of this trout population?

(b) What is the carrying capacity for this trout population?

(c) Sketch a rough graph of this trout population being sure to show any long-term behavior.

8. (12 points) Suppose y is a function of t which satisfies the differential equation

$$\frac{dy}{dt} = \frac{3(y-6)(y-24)}{20}$$

(a) For which values of y is the quantity y increasing?

(b) For which values of y is the quantity y decreasing?

(c) For which values of y is the quantity y in equilibrium? Determine whether each of these equilibrium values is stable or unstable.

9. (14 points) Suppose that 100 rabbits are released on an island that had no previous rabbits. Let R denote the rabbit population t months after they were released. The rabbit population grows at a rate which is proportional to the population size itself, where the constant of proportionality is 0.05 (i.e. a continuous growth rate of 5% per month).

   (a) Write down a differential equation with initial condition for the growth of this rabbit population.

   (b) Find a formula for R as a function of t.

   (c) Use your formula to determine the number of rabbits on the island 12 months after they were released.

10. (10 points) Find a formula for y as a function of t in the following initial value problems.

(a) $\dfrac{dy}{dt} = 6t^2 + 5, \qquad y(0) = 8$

(b) $\dfrac{dy}{dt} = \dfrac{2t}{3y^2}, \qquad y(0) = 5$

11. (12 points) Given the following initial value problem, use Euler's Method with $\Delta t = 2$ to estimate $y(6)$.

$$\frac{dy}{dt} = \sqrt{y}, \qquad y(0) = 10$$