Math 172 (Section 1) **Test 2** **October 24, 2005**

Name _____

1. (12 points) For which one of the following discrete dynamical systems is $u(n)$ oscillating toward its equilibrium value and getting 30% closer each day? (Assuming n is measured in days)

   (a) $u(n) = 1.3u(n-1) - 0.3$

   (b) $u(n) = -1.3u(n-1) + 0.3$

   (c) $u(n) = 0.7u(n-1) - 0.7$

   (d) $u(n) = -0.7u(n-1) + 0.3$

   (e) $u(n) = 0.3u(n-1) - 0.7$

   (f) $u(n) = -0.3u(n-1) + 0.7$

2. (11 points) Find an explicit expression that satisfies the discrete dynamical system with initial value shown.

$$u(n) = 0.7u(n-1) + 60$$
$$u(0) = 220$$

3. (15 points) A person currently has 4.8 mg of some toxin in his blood system, but from this day forward 4% of this toxin will be removed each day. Let $u(n)$ represent the number of milligrams of this toxin in the person's blood system n days from now.

   (a) A discrete dynamical system with initial condition for $u(n)$ is given by

   $u(n) =$

   $u(0 =$

   (b) An explicit formula for $u(n)$ is given by

   $u(n) =$

   (c) How long will it take until only 0.3 mg of this toxin will remain in the person's blood system?

4. (12 points) DataSource, a database management company, first went online on January 1, 2000. Their sales (in millions of dollars) are recorded at the end of each year since then in the table below.

year	2000	2001	2002	2003	2004
sales	12.1	14.2	17.4	24.2	29.7

(a) Let t represent the number of years since 2000, and then use the regression features of your calculator to find an exponential function which best fits this data. Each number from your formula should be correctly rounded to two places after the decimal point.

(b) According to your formula, by what percentage are sales increasing each year?

5. (12 points) Suppose a certain chemical is eliminated from the body by the kidneys and the liver. Let $u(n)$ represent the amount of this chemical in a person's bloodstream after n days. Assume that each day, the kidneys remove 20% of the chemical from the blood. Also assume that each day, the fraction of the chemical that is broken down by enzymes from the liver is given by

$$\frac{3}{4+u(n-1)}$$

Finally, assume that each day, the person takes a dose of 10 mg of this chemical. Develop a discrete dynamical system for $u(n)$. You do not need an initial value.

6. (16 points) The following four questions are based upon the discrete dynamical system

$$u(n) = 0.1u^2(n-1) + 0.3u(n-1) + 1$$

(a) Find the equilibrium values for this discrete dynamical system.

(b) For each equilibrium value, determine if it is stable or unstable.

(c) For each stable equilibrium value, determine the maximum interval of stability.

(d) For each stable equilibrium value, if the initial value is within the maximum interval of stability but different from the equilibrium value itself, then approximate the rate at which $u(n)$ goes toward the equilibrium value.

7. (10 points) Suppose that 50 wild pigs are released on an island. If the population of these pigs grows according to a logistic model with an intrinsic growth rate of 0.05 and a carrying capacity of 1000, then determine a discrete dynamical system with initial condition for this population of wild pigs.

8. (12 points) Suppose the metabolism of some person is such that the discrete dynamical system modeling the elimination of alcohol is

$$a(n) = a(n-1) - \frac{10a(n-1)}{4 + a(n-1)} + d$$

where $a(n)$ is the amount of alcohol (in grams) in the person's bloodstream after n hours of drinking d grams of alcohol per hour.

Suppose this person's weight is such that 40 grams of alcohol in the bloodstream represents a blood alcohol level of 0.08 (the amount in SC for a DWI conviction).

If this person drinks 3 cans of beer each hour for 3 hours, but then stops drinking for the next 3 hours, will his blood alcohol level fall below 40 grams? To justify your answer, you will need to fill in the remaining entries in the table below. Recall that each can of beer contains about 14 grams of alcohol.

Hint: Think carefully about the modifications needed to obtain the last 3 entries when he is no longer drinking.

n	a(n)
0	0
1	42.0
2	
3	
4	
5	
6	