1. $y = 5$, so $\dfrac{dy}{dx} = 0$

2. $y = 3x$, so $\dfrac{dy}{dx} = 3$

3. $y = 5x + 13$, so $\dfrac{dy}{dx} = 5$

4. $y = x^{12}$, so $\dfrac{dy}{dx} = 12x^{11}$

5. $y = x^{-12}$, so $\dfrac{dy}{dx} = -12x^{-13}$

6. $y = x^{4/3}$, so $\dfrac{dy}{dx} = \dfrac{4}{3}x^{1/3}$

7. $y = 8t^3$, so $\dfrac{dy}{dt} = 24t^2$

8. $y = 3t^4 - 2t^2$, so $\dfrac{dy}{dt} = 12t^3 - 4t$

9. $f(x) = \dfrac{1}{x^4}$ can be rewritten as $f(x) = x^{-4}$, so $f'(x) = -4x^{-5}$

10. $f(q) = q^3 + 10$, so $f'(q) = 3q^2$

11. $f(x) = Cx^2$, so $f'(x) = 2Cx$

12. $y = x^2 + 5x + 9$, so $\dfrac{dy}{dx} = 2x + 5$

13. $y = 6x^3 + 4x^2 - 2x$, so $\dfrac{dy}{dx} = 18x^2 + 8x - 2$

14. $y = -3x^4 - 4x^3 - 6x + 2$, so $\dfrac{dy}{dx} = -12x^3 - 12x^2 - 6$

15. $y = 3x^2 + 7x - 9$, so $\dfrac{dy}{dx} = 6x + 7$

16. $y = 8t^3 - 4t^2 + 12t - 3$, so $\dfrac{dy}{dt} = 24t^2 - 8t + 12$

17. $y = 4.2q^2 - 0.5q + 11.27$, so $\dfrac{dy}{dq} = 8.4q - 0.5$

18. $y = ax^2 + bx + c$, so $\dfrac{dy}{dx} = 2ax + b$

19. $y = z^2 + \dfrac{1}{2z}$ can be rewritten as $y = z^2 + \dfrac{1}{2}z^{-1}$, so $\dfrac{dy}{dz} = 2z - \dfrac{1}{2}z^{-2}$

20. $y = 3t^2 + \dfrac{12}{\sqrt{t}} - \dfrac{1}{t^2}$ can be rewritten as $y = 3t^2 + 12t^{-1/2} - t^{-2}$, so $\dfrac{dy}{dt} = 6t - 6t^{-3/2} + 2t^{-3}$

21. $y = 3t^5 - 5\sqrt{t} + \dfrac{7}{t}$ can be rewritten as $y = 3t^5 - 5t^{1/2} + 7t^{-1}$, so $\dfrac{dy}{dt} = 15t^4 - \dfrac{5}{2}t^{-1/2} - 7t^{-2}$

22. $f(x) = x^2 + 1$, so $f'(x) = 2x$. Thus $f'(0) = 0$, $f'(1) = 2$, $f'(2) = 4$, and $f'(-1) = -2$. You can also graph $y = x^2 + 1$ and use the built-in dy/dx button at $x = 0$, $x = 1$, $x = 2$, and $x = -1$.

27. $P(t) = t^3 + 4t + 1$, so $P'(t) = 3t^2 + 4$. Thus $P'(2) = 16$.

32. To get the equation of a line, we need a point and the slope. The point $(1,1)$ is given. $f(x) = 2x^3 - 2x^2 + 1$, so $f'(x) = 6x^2 - 4x$. Thus $f'(1) = 2$ so we know the slope at $x = 1$ is equal to 2. We use the point-slope formula $y - y_1 = m(x - x_1)$ to obtain $y - 1 = 2(x - 1)$ which simplifies to our answer $y = 2x - 1$. Try graphing $f(x) = 2x^3 - 2x^2 + 1$ and $y = 2x - 1$ to see that the line really is tangent to the graph at $(1,1)$.

33. $f(t) = 6t - t^2$, so $f'(t) = 6 - 2t$. Thus $f'(4) = -2$ and we know that the slope is -2. Since $f(4) = 8$, our point is $(4,8)$. Now the point-slope formula gives $y - 8 = -2(t - 4)$ which simplifies to our answer $y = -2t + 16$.

34. $C(q) = 1000 + 2q^2$, so $C'(q) = 4q$. Thus $C'(25) = 100$ dollars per item, so the marginal cost of producing the 25th item is \$100.

36. $Z(t) = 300t^2$, so $Z'(t) = 600t$. Thus $Z(4) = 4800$ and $Z'(4) = 2400$. After 4 months, there are 4800 mussels in the bay and they are growing at a rate of 2400 mussels per month.

37. $f(x) = 320 + 140x - 10x^2$, so $f'(x) = 140 - 20x$. Thus $f(5) = 770$ and $f'(5) = 40$. If 5 pounds of fertilizer are used on each acre, then the yield on each acre is 770 bushels and increasing at a rate of 40 bushels per pound. You should use more fertilizer.

43. $y = 1250 - 16t^2$, so velocity $= \dfrac{dy}{dt} = -32t$. The height of the ball is decreasing as it falls so the sign of the velocity, as expected, is negative. The height of the ball is 0 when it hits the ground. Now solving $0 = 1250 - 16t^2$, we get that $t \approx \pm 8.8388$. Only the positive value makes sense here. Plugging this into the velocity equation gives $\dfrac{dy}{dt} \approx -282.84$ feet per second. This converts to about -192.85 miles per hour.

1. $y = 5t^2 + 4e^t$, so $\dfrac{dy}{dt} = 10t + 4e^t$
2. $f(x) = 2e^x + x^2$, so $f'(x) = 2e^x + 2x$
3. $f(x) = 2^x + 2 \cdot 3^x$, so $f'(x) = (\ln 2)2^x + 2(\ln 3)3^x$
4. $y = 4 \cdot 10^x - x^3$, so $\dfrac{dy}{dx} = 4(\ln 10)10^x - 3x^2$
5. $y = 3x - 2 \cdot 4^x$, so $\dfrac{dy}{dx} = 3 - 2(\ln 4)4^x$
6. $y = \dfrac{3^x}{3} + \dfrac{33}{\sqrt{x}}$ can be rewritten as $y = \dfrac{1}{3} \cdot 3^x + 33x^{-1/2}$, so $\dfrac{dy}{dx} = \dfrac{1}{3}(\ln 3)3^x - \dfrac{33}{2}x^{-3/2}$
7. $f(x) = x^3 + 3^x$, so $f'(x) = 3x^2 + (\ln 3)3^x$
8. $y = 5 \cdot 5^t + 6 \cdot 6^t$, so $\dfrac{dy}{dt} = 5(\ln 5)5^t + 6(\ln 6)6^t$
9. $P(t) = Ce^t$, so $P'(t) = Ce^t$
10. $D = 10 - \ln p$, so $\dfrac{dD}{dp} = -\dfrac{1}{p}$
11. $R = 3 \ln q$, so $\dfrac{dR}{dq} = \dfrac{3}{q}$
12. $y = t^2 + 5 \ln t$, so $\dfrac{dy}{dt} = 2t + \dfrac{5}{t}$
13. $y = B + Ae^t$, so $\dfrac{dy}{dt} = Ae^t$
14. $f(x) = Ae^x - Bx^2 + C$, so $f'(x) = Ae^x - 2Bx$
15. $P = 3t^3 + 2e^t$, so $\dfrac{dP}{dt} = 9t^2 + 2e^t$
16. $P(t) = 3000(1.02)^t$, so $P'(t) = 3000(\ln 1.02)1.02^t$
17. $P(t) = 12.41(0.94)^t$, so $P'(t) = 12.41(\ln 0.94)0.94^t$
18. $y = 5(2^x) - 5x + 4$, so $\dfrac{dy}{dx} = 5(\ln 2)2^x - 5$
19. $R(q) = q^2 - 2 \ln q$, so $R'(q) = 2q - \dfrac{2}{q}$
20. $y = x^2 + 4x - 3 \ln x$, so $\dfrac{dy}{dx} = 2x + 4 - \dfrac{3}{x}$

21. $f(t) = Ae^t + B\ln t$, so $f'(t) = Ae^t + \dfrac{B}{t}$

23. $y = 3^x$, so $\dfrac{dy}{dx} = (\ln 3)3^x$. At $x = 1$, we get $y = 3$ and $\dfrac{dy}{dx} = 3\ln 3$. So our point is $(1, 3)$ and the slope is $3\ln 3$. Now the point-slope formula gives $y - 3 = 3\ln 3(x-1)$ which simplifies to our answer $y = 3\ln 3x + 3 - 3\ln 3$. Thus $y \approx 3.3x - 0.3$.

27. $P = 35,000(0.98)^t$, so $\dfrac{dP}{dt} = 35,000(\ln 0.98)(0.98)^t$. So at $t = 23$, we get $\dfrac{dP}{dt} \approx -444.3$. So on January 1, 1983, the population is decreasing by approximately 444.3 people per year.

30. $P = 10.8(0.998)^t$, so $\dfrac{dP}{dt} = 10.8(\ln 0.998)(0.998)^t$. Thus at $t = 10$, we have $P \approx 10.58593367$ and $\dfrac{dP}{dt} \approx -0.0211930675$. So in the year 2000, the population is predicted to be 10,585,934 and decreasing by 21,193 people per year.

32. $y = \ln x$, so $\dfrac{dy}{dx} = \dfrac{1}{x}$. At $x = 1$, we get $y = 0$ and $\dfrac{dy}{dx} = 1$. So our point is $(1, 0)$ and the slope is 1. Now the point-slope formula gives $y - 0 = 1 \cdot (x - 1)$ which simplifies to our answer $y = x - 1$. Since the original function and the tangent line look similar near the point of tangency, we get that $\ln x \approx x - 1$ for x near 1. For instance, $\ln 1.1 \approx 1.1 - 1 = 0.1$, and $\ln 2 \approx 2 - 1 = 1$. Compare these to the values given by your calculator. Use the graphs of $y = \ln x$ and $y = x - 1$ to explain why one of the approximations is better.

37. $V(t) = 25(0.85)^t$, so $V(4) \approx 13.050$. This means that the value of the automobile is about \$13,050 in 2001. $V'(t) = 25(\ln 0.85)(0.85)^t$ thousand dollars per year, so $V'(4) \approx -2.121$. So in 2001, the value of the car is going down by about 2,121 dollars per year.

1. $f(x) = (x+1)^{99}$, so $f'(x) = 99(x+1)^{98} \cdot (1) = 99(x+1)^{98}$
2. $R = (q^2+1)^4$, so $\dfrac{dR}{dq} = 4(q^2+1)^3 \cdot (2q) = 8q(q^2+1)^3$
3. $w = (t^2+1)^{100}$, so $\dfrac{dw}{dt} = 100(t^2+1)^{99} \cdot (2t) = 200t(t^2+1)^{99}$
4. $w = (t^3+1)^{100}$, so $\dfrac{dw}{dt} = 100(t^3+1)^{99} \cdot (3t^2) = 300t^2(t^3+1)^{99}$
5. $w = (5r-6)^3$, so $\dfrac{dw}{dr} = 3(5r-6)^2 \cdot (5) = 15(5r-6)^2$
6. $f(t) = e^{3t}$, so $f'(t) = e^{3t} \cdot (3) = 3e^{3t}$
7. $y = e^{0.7t}$, so $\dfrac{dy}{dt} = e^{0.7t} \cdot (0.7) = 0.7e^{0.7t}$
8. $y = e^{-4t}$, so $\dfrac{dy}{dt} = e^{-4t} \cdot (-4) = -4e^{-4t}$
9. $y = \sqrt{s^3+1}$ can be rewritten as $y = (s^3+1)^{1/2}$, so $\dfrac{dy}{ds} = \dfrac{1}{2}(s^3+1)^{-1/2} \cdot (3s^2) = \dfrac{3s^2}{2\sqrt{s^3+1}}$
10. $w = e^{\sqrt{s}}$, so $\dfrac{dw}{ds} = e^{\sqrt{s}} \cdot \left(\dfrac{1}{2}s^{-1/2}\right) = \dfrac{e^{\sqrt{s}}}{2\sqrt{s}}$ $\left(\text{We used that } \dfrac{d}{ds}(\sqrt{s}) = \dfrac{1}{2}s^{-1/2}\right)$
11. $P = e^{-0.2t}$, so $\dfrac{dP}{dt} = e^{-0.2t} \cdot (-0.2) = -0.2e^{-0.2t}$
12. $w = e^{-3t^2}$, so $\dfrac{dw}{dt} = e^{-3t^2} \cdot (-6t) = -6te^{-3t^2}$
13. $y = \ln(5t+1)$, so $\dfrac{dy}{dt} = \dfrac{1}{5t+1} \cdot (5) = \dfrac{5}{5t+1}$
14. $P = 50e^{-0.6t}$, so $\dfrac{dP}{dt} = 50e^{-0.6t} \cdot (-0.6) = -30e^{-0.6t}$
15. $P = 200e^{0.12t}$, so $\dfrac{dP}{dt} = 200e^{0.12t} \cdot (0.12) = 24e^{0.12t}$
16. $y = 12 - 3x^2 + 2e^{3x}$, so $\dfrac{dy}{dx} = -6x + 2e^{3x} \cdot (3) = -6x + 6e^{3x}$
17. $C = 12(3q^2-5)^3$, so $\dfrac{dC}{dq} = 36(3q^2-5)^2 \cdot (6q) = 216q(3q^2-5)^2$
18. $f(x) = 6e^{5x} + e^{-x^2}$, so $f'(x) = 6e^{5x} \cdot (5) + e^{-x^2} \cdot (-2x) = 30e^{5x} - 2xe^{-x^2}$
19. $y = 5e^{5t+1}$, so $\dfrac{dy}{dt} = 5e^{5t+1} \cdot (5) = 25e^{5t+1}$

20. $f(x) = \ln(1-x)$, so $f'(x) = \dfrac{1}{1-x} \cdot (-1) = \dfrac{-1}{1-x}$

21. $f(t) = \ln(t^2+1)$, so $f'(t) = \dfrac{1}{t^2+1} \cdot (2t) = \dfrac{2t}{t^2+1}$

22. $f(x) = \ln(1-e^{-x})$, so $f'(x) = \dfrac{1}{1-e^{-x}} \cdot (0 - e^{-x} \cdot (-1)) = \dfrac{e^{-x}}{1-e^{-x}}$

23. $f(x) = \ln(e^x+1)$, so $f'(x) = \dfrac{1}{e^x+1} \cdot (e^x) = \dfrac{e^x}{e^x+1}$

24. $f(t) = 5\ln(5t+1)$, so $f'(t) = 5\left(\dfrac{1}{5t+1}\right) \cdot (5) = \dfrac{25}{5t+1}$

25. $g(t) = \ln(4t+9)$, so $g'(t) = \dfrac{1}{4t+9} \cdot (4) = \dfrac{4}{4t+9}$

26. $y = 5 + \ln(3t+2)$, so $\dfrac{dy}{dt} = \dfrac{1}{3t+2} \cdot (3) = \dfrac{3}{3t+2}$

27. $Q = 100(t^2+5)^{0.5}$, so $\dfrac{dQ}{dt} = 50(t^2+5)^{-0.5} \cdot (2t) = 100t(t^2+5)^{-0.5}$

28. $y = 5x + \ln(x+2)$, so $\dfrac{dy}{dx} = 5 + \dfrac{1}{x+2} \cdot (1) = 5 + \dfrac{1}{x+2}$

29. $y = (5+e^x)^2$, so $\dfrac{dy}{dx} = 2(5+e^x) \cdot (e^x) = 2e^x(5+e^x)$

30. $P = (1+\ln x)^{0.5}$, so $\dfrac{dP}{dx} = 0.5(1+\ln x)^{-0.5} \cdot \left(\dfrac{1}{x}\right)$

31. $y = e^{-2t}$, so $\dfrac{dy}{dt} = e^{-2t} \cdot (-2)$. At $t = 0$, we get $y = 1$ and $\dfrac{dy}{dt} = -2$. So our point is $(0,1)$ and the slope is -2. Now the point-slope formula gives $y - 1 = -2(t-0)$ which simplifies to our answer $y = -2t + 1$.

34. $C(q) = 1000 + 30e^{0.05q}$, so $C'(q) = 30e^{0.05q} \cdot (0.05)$. We let $q = 50$ to get the cost of \$1365.47 and marginal cost of 18.27 dollars per item.

35. $f(t) = 5.3e^{0.018t}$, so $f'(t) = 5.3e^{0.018t} \cdot (0.018)$. We get $f(0) = 5.3$ and $f'(0) = 0.0954$. This says that in 1990, the population was 5.3 billion and growing at a rate of 0.0954 billion people per year. Letting $t = 10$, we get $f(10) \approx 6.3$ and $f'(10) \approx 0.1142$. This says that in 2000, the population was predicted to be approximately 6.3 billion and growing at a rate of 0.1142 billion people per year.

37. $H = 40 + 30e^{-2t}$, so $\dfrac{dH}{dt} = 30e^{-2t} \cdot (-2) = -60e^{-2t}$. The sign of $\dfrac{dH}{dt}$ is negative, as expected, since the temperature is decreasing. The magnitude of $\dfrac{dH}{dt}$ is largest at $t = 0$ since the temperature of the can of soda will decrease most rapidly right when you put it in the refrigerator.

3. $f(x) = xe^x$, so $f'(x) = (1) \cdot (e^x) + (x) \cdot (e^x)$
4. $f(t) = te^{-2t}$, so $f'(t) = (1) \cdot \left(e^{-2t}\right) + (t) \cdot \left(-2e^{-2t}\right)$
5. $y = x \cdot 2^x$ so $\dfrac{dy}{dx} = (1) \cdot (2^x) + (x) \cdot ((\ln 2)\, 2^x)$
6. $y = 5xe^{x^2}$, so $\dfrac{dy}{dx} = (5) \cdot \left(e^{x^2}\right) + (5x) \cdot \left(2xe^{x^2}\right)$
7. $y = t^2(3t+1)^3$, so $\dfrac{dy}{dt} = (2t) \cdot \left((3t+1)^3\right) + \left(t^2\right) \cdot \left(9(3t+1)^2\right)$
8. $y = x \ln x$, so $\dfrac{dy}{dx} = (1) \cdot (\ln x) + (x) \cdot \left(\dfrac{1}{x}\right)$
9. $w = (t^3+5t)(t^2-7t+2)$, so $\dfrac{dw}{dt} = \left(3t^2+5\right) \cdot \left(t^2-7t+2\right) + \left(t^3+5t\right) \cdot (2t-7)$
10. $y = (t^2+3)e^t$, so $\dfrac{dy}{dt} = (2t) \cdot \left(e^t\right) + \left(t^2+3\right) \cdot \left(e^t\right)$
11. $z = (3t+1)(5t+2)$, so $\dfrac{dz}{dt} = (3) \cdot (5t+2) + (3t+1) \cdot (5)$
12. $y = (t^3-7t^2+1)e^t$, so $\dfrac{dy}{dt} = \left(3t^2-14t\right) \cdot \left(e^t\right) + \left(t^3-7t^2+1\right) \cdot \left(e^t\right)$
13. $P = t^2 \ln t$, so $\dfrac{dP}{dt} = (2t) \cdot (\ln t) + \left(t^2\right) \cdot \left(\dfrac{1}{t}\right)$
14. $f(x) = \dfrac{x^2+3}{x}$, so $f'(x) = \dfrac{(2x) \cdot (x) - (x^2+3) \cdot (1)}{(x)^2}$
15. $R = 3qe^{-q}$, so $\dfrac{dR}{dq} = (3) \cdot \left(e^{-q}\right) + (3q) \cdot \left(-e^{-q}\right)$
16. $y = te^{-t^2}$, so $\dfrac{dy}{dt} = (1) \cdot \left(e^{-t^2}\right) + (t) \cdot \left(-2te^{-t^2}\right)$
17. $f(z) = \sqrt{z}e^{-z}$, so $f'(z) = \left(\dfrac{1}{2}z^{-1/2}\right) \cdot \left(e^{-z}\right) + \left(\sqrt{z}\right) \cdot \left(-e^{-z}\right)$
18. $g(p) = p\ln(2p+1)$, so $g'(p) = (1) \cdot (\ln(2p+1)) + (p) \cdot \left(\dfrac{2}{2p+1}\right)$
19. $f(t) = te^{5-2t}$, so $f'(t) = (1) \cdot \left(e^{5-2t}\right) + (t) \cdot \left(-2e^{5-2t}\right)$
20. $f(w) = (5w^2+3)e^{w^2}$, so $f'(w) = (10w) \cdot \left(e^{w^2}\right) + \left(5w^2+3\right) \cdot \left(2we^{w^2}\right)$

21. $f(x) = \dfrac{x}{e^x}$, so $f'(x) = \dfrac{(1) \cdot (e^x) - (x) \cdot (e^x)}{(e^x)^2}$

22. $w = \dfrac{3z}{1+2z}$, so $\dfrac{dw}{dz} = \dfrac{(3) \cdot (1+2z) - (3z) \cdot (2)}{(1+2z)^2}$

23. $z = \dfrac{1-t}{1+t}$, so $\dfrac{dz}{dt} = \dfrac{(-1) \cdot (1+t) - (1-t) \cdot (1)}{(1+t)^2}$

24. $y = \dfrac{e^x}{1+e^x}$, so $\dfrac{dy}{dx} = \dfrac{(e^x) \cdot (1+e^x) - (e^x) \cdot (e^x)}{(1+e^x)^2}$

25. $w = \dfrac{3y+y^2}{5+y}$, so $\dfrac{dw}{dy} = \dfrac{(3+2y) \cdot (5+y) - (3y+y^2) \cdot (1)}{(5+y)^2}$

26. $y = \dfrac{1+z}{\ln z}$, so $\dfrac{dy}{dz} = \dfrac{(1) \cdot (\ln z) - (1+z) \cdot \left(\frac{1}{z}\right)}{(\ln z)^2}$

29. $f(t) = 20te^{-0.04t}$, so $f'(t) = (20) \cdot (e^{-0.04t}) + (20t) \cdot (-0.04e^{-0.04t})$. For part (a), you should sketch the graph of $f(t)$. From your graph, you see that $f(t)$ is increasing at $t = 15$ and decreasing at $t = 45$. Therefore, $f'(15)$ is positive and $f'(45)$ is negative. For part (b), you are asked to evaluate $f(30)$ and $f'(30)$ analytically. Using the formulas above, we get $f(30) \approx 180.7$ and $f'(30) \approx -1.2$. This means that 30 minutes after the drug was taken, the concentration in the bloodstream is 180.7 mg/ml. The concentration at that time is decreasing by 1.2 mg/ml per minute.

30. $P(t) = t \ln t$, so $P'(t) = (1) \cdot (\ln t) + (t) \cdot \left(\frac{1}{t}\right)$. $P(2) = 2 \ln 2$ so we have the point $(2, 2\ln 2)$. $P'(2) = \ln 2 + 1$ so we have a slope of $\ln 2 + 1$. Using the point-slope formula, we get the equation of the tangent line to be $y - 2\ln 2 = (\ln 2 + 1)(t-2)$ which can be rewritten as $y = (\ln 2 + 1)t - 2$ or $y \approx 1.69t - 2$.