To answer some of these problems, you should be familiar with the following Maclaurin series.

- $\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + \cdots$ for $-1 < x < 1$
- $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$ for $-1 < x \le 1$
- $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$ for $-1 \le x \le 1$
- $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$ for $-\infty < x < \infty$
- $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$ for $-\infty < x < \infty$
- $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$ for $-\infty < x < \infty$

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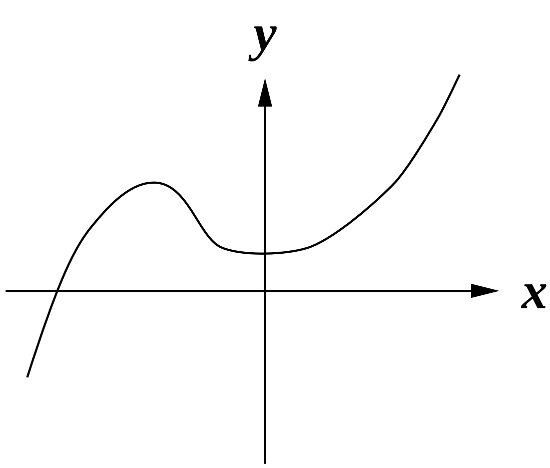
1. Let $P_0(x)$, $P_1(x)$, and $P_2(x)$ be the Maclaurin polynomials of order 0, 1, and 2, respectively, for the function graphed below.



   (a) Sketch $P_0(x)$ and $P_1(x)$ and be sure to label which one is P_0 and which is P_1.

   (b) Let $P_2(x) = a_0 + a_1 x + a_2 x^2$.

      i. Is a_0 positive, negative, or zero? Explain.

      ii. Is a_1 positive, negative, or zero? Explain.

      iii. Is a_2 positive, negative, or zero? Explain.

2. Use Maclaurin polynomials for $\cos x$ to approximate $\cos 1$. What can you determine about the error in your approximation?

3. Find the Maclaurin series for $f(x) = 4\arctan x$. What interesting fact do you learn when using the series to approximate $f(1)$?

4. Approximate $\ln(1.1)$ with an error of less than 0.001 by looking at Taylor polynomials based at 1 for $\ln x$. Explain how you know that your error is less than 0.001.

5. Use an appropriate Taylor polynomial of order 2 to obtain an estimate for $\sqrt[3]{1.3}$.

6. Use the most appropriate Taylor polynomial of order 2 to obtain an estimate for $\sqrt{101}$.

7. Determine whether the following series converge or diverge. Find the exact sum of those series which converge. To find the exact sum, it may help to think about some of the important Maclaurin series.

   (a) $1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots$

   (b) $1 + 2 + \frac{2^2}{2!} + \frac{2^3}{3!} + \frac{2^4}{4!} + \cdots$

   (c) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$

   (d) $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$

   (e) $1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \cdots$

   (f) $1 + 2(0.5) + 3(0.5)^2 + 4(0.5)^3 + 5(0.5)^4 + \cdots$

   (g) $\frac{1}{3} + \frac{2}{3^2} + \frac{3}{3^3} + \frac{4}{3^4} + \frac{5}{3^5} + \cdots$

8. Find the exact sum of the series $\sum_{k=2}^{\infty} k(k-1)x^{k-2}$ by recognizing how it is related to something familiar. For which values of x is this sum valid?

9. Use Maclaurin series to approximate the following definite integrals. Give an upper bound on the error in your approximation.

   (a) $\int_0^1 e^{-x^2}\,dx$

   (b) $\int_0^1 \sin\left(x^2\right)\,dx$

   (c) $\int_0^1 \frac{8}{8 + x^3}\,dx$

10. Use Maclaurin series to evaluate the following limits.

   (a) $\lim_{x \to 0} \frac{\sin x}{x}$

   (b) $\lim_{x \to 0} \frac{\sin x - x + x^3/6}{x^5}$

   (c) $\lim_{x \to 0} \frac{\tan^{-1} x - x}{x^3}$

11. Use any short-cut methods learned in class or in section 10.10 to find the Maclaurin series for the following functions. You should list at least the first 4 nonzero terms in each series and state the interval of convergence.

   (a) $\frac{1}{1+2x}$

   (b) $\frac{3x}{1-x^2}$

   (c) $x \sin\left(x^2\right)$

   (d) $f(x) = \frac{1+x}{1-x}$

   (e) $f(x) = x^2 e^{-x}$

   (f) $f(x) = \frac{x}{(1-x)(1-2x)}$

   (g) e^{-2x}

   (h) e^{2+x}

   (i) $\sqrt{e^x}$

   (j) $\frac{6}{2+x}$

   (k) $\ln\left((2+2x)^5\right)$

   (l) $\ln\left(\frac{1-x}{1+x}\right)$

   (m) $\frac{5x-1}{x^2-1}$

   (n) $\frac{1}{(1-x)^2}$

   (o) $e^x \sin x$

   (p) $e^{x^2} \cos x$

   (q) $6 \sin x \cos x$

   (r) $\frac{\tan^{-1} x}{1+x}$

   (s) $\frac{\ln(1+x)}{1-x}$