Math 142 (Section 501) **Quiz 11 Solutions** **Spring 2008**

Determine if the following series converge or diverge. You must fully justify your answer.

1. $\displaystyle\sum_{k=3}^{\infty} \frac{5^k}{k!}$

Since this series has positive terms, we may apply the ratio test.

$$\begin{aligned}
\rho &= \lim_{n\to\infty} \frac{a_{n+1}}{a_n} \\
&= \lim_{n\to\infty} \frac{5^{n+1}/(n+1)!}{5^n/n!} \\
&= \lim_{n\to\infty} \frac{n!5^{n+1}}{(n+1)!5^n} \\
&= \lim_{n\to\infty} \frac{5}{n+1} \\
&= 0
\end{aligned}$$

The series converges since $\rho < 1$.

2. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{(3+\ln k)^k}$

Since this series has positive terms, we may apply the root test.

$$\begin{aligned}
\rho &= \lim_{n\to\infty} \sqrt[n]{\frac{1}{(3+\ln n)^n}} \\
&= \lim_{n\to\infty} \frac{1}{3+\ln n} \\
&= 0
\end{aligned}$$

The series converges since $\rho < 1$.

3. $\displaystyle\sum_{k=5}^{\infty} \frac{1}{2+\sqrt{k}}$

This series and the series $\displaystyle\sum_{k=5}^{\infty} \frac{1}{\sqrt{k}}$ both have all positive terms, so we may apply the limit comparison test.

$$\begin{aligned}
\rho &= \lim_{n\to\infty} \frac{a_n}{b_n} \\
&= \lim_{n\to\infty} \frac{1/\sqrt{n}}{1/(2+\sqrt{n})} \\
&= \lim_{n\to\infty} \frac{2+\sqrt{n}}{\sqrt{n}} \\
&= \lim_{n\to\infty} \left(\frac{2}{\sqrt{n}}+1\right) \\
&= 1
\end{aligned}$$

Since ρ is positive and finite, either both series converge or both series diverge. Since $\displaystyle\sum_{k=5}^{\infty} \frac{1}{\sqrt{k}}$ is a divergent p-series ($p = 1/2 < 1$), we have that the original series also diverges.