

Name _____

1. (10 points) Complete the list below by writing down the Maclaurin series and the interval of convergence for each function.

- $\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + \dots$ for $-1 < x < 1$

- $\sin x =$

- $\cos x =$

- $\tan^{-1} x =$

- $\ln(1+x) =$

- $e^x =$

2. (6 points) Find the Maclaurin series and the interval of convergence for $f(x) = \frac{x^3}{1 - 2x}$.

3. (6 points) Use an appropriate Maclaurin series to approximate the following definite integral. You do not need to discuss the error term, but you should use at least 3 non-zero terms in the series for your approximation.

$$\int_0^{1/2} \frac{1}{1 + x^3} dx$$

4. (5 points) Approximate the quantity $\frac{1}{\sqrt{e}}$ with an error of less than 0.1 by using an appropriate Maclaurin series. Specifically state why you know that the error is less than 0.1.

5. (5 points) Find the 3rd Taylor polynomial for $f(x) = \sqrt{x}$ about $x = 4$.

6. (6 points) Find the interval of convergence for the following power series. Thoroughly justify your answer.

$$\sum_{k=0}^{\infty} \left(\frac{9}{4}\right)^k x^{2k}$$

7. (6 points) Find the interval of convergence for the following power series. Thoroughly justify your answer.

$$\sum_{k=1}^{\infty} \frac{(x-2)^k}{k}$$

8. (6 points each) Determine if the following series converge or diverge. You must thoroughly justify each answer. You do not need to compute the sums of the convergent series.

(a) $\sum_{k=1}^{\infty} \frac{1}{2k}$

(b) $\sum_{k=0}^{\infty} \frac{2^{2k+1}}{3^k}$

$$(c) \sum_{k=1}^{\infty} \frac{k!}{2^k}$$

$$(d) \sum_{k=0}^{\infty} \frac{(-1)^k}{e^k}$$

$$(e) \sum_{k=1}^{\infty} \frac{k}{2k+1}$$

$$(f) \sum_{k=2}^{\infty} \frac{1}{k^2+1}$$

$$(g) \sum_{k=2}^{\infty} \frac{k}{k^2 + 1}$$

$$(h) \sum_{k=2}^{\infty} \frac{k^2}{k^2 + 1}$$

9. (2 points each) Find the exact sum for the following convergent series and simplify your answer. You do not need to justify why the series are convergent. For some of these it may help to work from the Maclaurin series that you already have memorized.

(a)
$$\sum_{k=1}^{\infty} \frac{3}{2^{k+1}}$$

(b)
$$\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}$$

$$(c) \sum_{k=1}^{\infty} \frac{4}{k(k+2)}$$

$$(d) \sum_{k=1}^{\infty} \frac{k+1}{2^k}$$