Math 142 **Practice Problems III (Series)**

To answer these problems, you should be familiar with the following Maclaurin series.

- $\dfrac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + \cdots \qquad \text{for } -1 < x < 1$
- $\ln(1+x) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} + \cdots \qquad \text{for } -1 < x \leq 1$
- $\tan^{-1} x = x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + \cdots \qquad \text{for } -1 \leq x \leq 1$
- $e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \dfrac{x^4}{4!} + \cdots \qquad \text{for } -\infty < x < \infty$
- $\sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \cdots \qquad \text{for } -\infty < x < \infty$
- $\cos x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \cdots \qquad \text{for } -\infty < x < \infty$

1. Use any short-cut methods learned in class or in section 10.10 to find the Maclaurin series for the following functions. You should list at least the first 4 nonzero terms in each series and state the interval of convergence.

   (a) $\dfrac{1}{1+2x}$

   (b) $\dfrac{3x}{1-x^2}$

   (c) $x \sin\left(x^2\right)$

   (d) e^{-2x}

   (e) e^{2+x}

   (f) $\sqrt{e^x}$

   (g) $\dfrac{6}{2+x}$

   (h) $\ln\left((2+2x)^5\right)$

   (i) $\ln\left(\dfrac{1-x}{1+x}\right)$

   (j) $\dfrac{5x-1}{x^2-1}$

(k) $\dfrac{1}{(1-x)^2}$

(l) $e^x \sin x$

(m) $e^{x^2} \cos x$

(n) $6 \sin x \cos x$

(o) $\dfrac{\tan^{-1} x}{1+x}$

(p) $\dfrac{\ln(1+x)}{1-x}$

2. Use Maclaurin series to approximate the following definite integrals. Give an upper bound on the error in your approximation.

   (a) $\displaystyle\int_0^1 e^{-x^2}\, dx$

   (b) $\displaystyle\int_0^1 \sin\left(x^2\right) dx$

   (c) $\displaystyle\int_0^1 \frac{8}{8+x^3}\, dx$

3. Use Maclaurin series to evaluate the following limits.

   (a) $\displaystyle\lim_{x\to 0} \frac{\sin x}{x}$

   (b) $\displaystyle\lim_{x\to 0} \frac{\sin x - x + x^3/6}{x^5}$

   (c) $\displaystyle\lim_{x\to 0} \frac{\tan^{-1} x - x}{x^3}$