Math 142 **Practice Problems II (Series)**

These are problems that I've asked on quizzes and tests from previous semesters.

1. Write down the Maclaurin series for each of the following functions and specify the values of x for which the series converge.

   (a) $\dfrac{1}{1-x}$

   (b) $\ln(1+x)$

   (c) $\tan^{-1} x$

   (d) e^x

   (e) $\sin x$

   (f) $\cos x$

   (g) $(1+x)^p$

2. Find the terms through x^6 in the Maclaurin series for $f(x) = x^2 e^{-x}$. Also specify its interval of convergence.

3. Find the terms through x^6 in the Maclaurin series for $f(x) = \dfrac{x}{(1-x)(1-2x)}$. Also specify its interval of convergence.

4. Find the exact sum of the series $\sum_{k=2}^{\infty} k(k-1)x^{k-2}$ by recognizing how it is related to something familiar. For which values of x is this sum valid?

5. Find the Maclaurin polynomial of order 3 for $f(x) = \arctan x$. Use this to approximate $f(1)$. You do not need to discuss the error term.

6. Approximate $\ln(1.1)$ with an error of less than 0.001 by looking at Taylor polynomials based at 1 for $\ln x$. Explain how you know that your error is less than 0.001.

7. There is a rule of thumb that for $a > 0$, $\sqrt{a^2+1} \approx a + \dfrac{1}{2a}$.

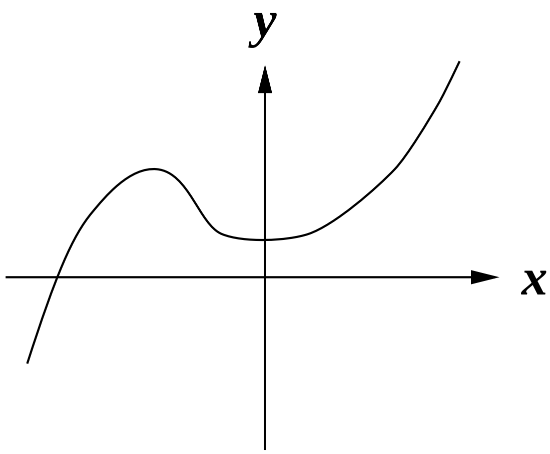
   (a) Since $26 = 5^2 + 1$, use the rule of thumb to approximate $\sqrt{26}$. Now approximate $\sqrt{101}$ and $\sqrt{10001}$.

   (b) Explain where the rule of thumb comes from by looking at Maclaurin polynomials for $\sqrt{a^2+x}$.

   (c) Use the error term for the Maclaurin polynomial used in part (b) to explain whether the rule of thumb works better when a is large or when a is small.

   (d) Come up with your own rule of thumb for approximating $\sqrt{a^2+2}$.

8. Let $P_0(x)$, $P_1(x)$, and $P_2(x)$ be the Maclaurin polynomials of order 0, 1, and 2, respectively, for the function graphed below.



(a) Sketch $P_0(x)$ and $P_1(x)$ and be sure to label which one is P_0 and which is P_1.

(b) Let $P_2(x) = a_0 + a_1x + a_2x^2$.

i. Is a_0 positive, negative, or zero? Explain.

ii. Is a_1 positive, negative, or zero? Explain.

iii. Is a_2 positive, negative, or zero? Explain.

9. Use an appropriate Taylor polynomial of order 2 to obtain an estimate for $\sqrt[3]{1.3}$.

10. Use the most appropriate Taylor polynomial of order 2 to obtain an estimate for $\sqrt{101}$. You do not need to discuss the error term.

11. Find the Maclaurin polynomial of order 4 for $\cos x$. Use this to approximate $\cos 1$. Without using your calculator, give a good bound for the error in your approximation.

12. By looking at the Taylor polynomials for an appropropriate function and discussing the error term **very carefully**, show that the infinite series $1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots$ converges to e.

13. Determine whether the following series converge or diverge. Find the exact sum of those series which converge. To find the exact sum, it may help to think about some of the important Maclaurin series.

(a) $1 + 2 + \frac{2^2}{2!} + \frac{2^3}{3!} + \frac{2^4}{4!} + \cdots$

(b) $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$

(c) $1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \cdots$

(d) $1 + 2(0.5) + 3(0.5)^2 + 4(0.5)^3 + 5(0.5)^4 + \cdots$

14. Find the terms through x^5 in the Maclaurin series for $f(x) = \dfrac{1+x}{1-x}$.

15. The **Rearrangement Theorem** states that you can rearrange the terms of a particular type of series without affecting either the convergence or the sum of the series. Which type of series is this theorem referring to? Circle the one correct choice.

(a) The alternating harmonic series.

(b) Any geometric series.

(c) Any absolutely convergent series.

(d) Any collapsing series.

(e) Any conditionally convergent series.

16. For a particular series $\sum_{k=1}^{\infty} a_k$, you are given the following information:

- $\sum_{k=1}^{\infty} a_k = \ln(2)$
- $\left|\sum_{k=1}^{\infty} a_k\right| = \ln(2)$
- $\sum_{k=1}^{\infty} |a_k|$ diverges

Which one of the following is a correct statement?

(a) $\sum_{k=1}^{\infty} a_k$ converges absolutely

(b) $\sum_{k=1}^{\infty} a_k$ converges conditionally

(c) $\sum_{k=1}^{\infty} a_k$ diverges

17. The following infinite series converges to a sum S.

$$S = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k^2} = 1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25} - \frac{1}{36} + \cdots$$

(a) Explicitly show that the two conditions necessary for an alternating series to converge are met here. Do not just say that the conditions obviously hold (as we have occasionally done in lecture).

(b) If we approximate the sum of the infinite series with

$$S \approx \sum_{k=1}^{4} (-1)^{k+1} \frac{1}{k^2} = 1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16},$$

then how can the error in our approximation be bounded? (fill in the blank below)

$|\text{error}| \leq$

(c) How many of the beginning terms of the infinite series could you add together to get an estimate for its sum S that was within 0.01 of the correct sum?

18. Find the convergence set for the following power series. You must thoroughly justify your claim.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{(n+2)3^n}$

(b) $1 + \dfrac{x^2}{2!} + \dfrac{x^4}{4!} + \dfrac{x^6}{6!} + \dfrac{x^8}{8!} + \cdots$