Math 142 (Section 502) **Quiz 9 Solutions** **Spring 2007**

1. Evaluate the following integral.

$$\int \left(\frac{1}{2x+5} + \frac{8}{(x+2)^3} \right) dx = \frac{1}{2} \ln(2x+5) - \frac{4}{(x+2)^2} + C$$

2. Using the constants $A, B, C, D, \ldots$, show the form of the partial fraction decomposition for the following rational function. Do not solve for the constants and do not evaluate an integral.

$$\frac{2x+5}{(x+2)(x-1)^3(x^2+1)^2} =$$

$$\frac{A}{x+2} + \frac{B}{x-1} + \frac{C}{(x-1)^2} + \frac{D}{(x-1)^3} + \frac{Ex+F}{x^2+1} + \frac{Gx+H}{(x^2+1)^2}$$

3. Evaluate the following integral.

$$\int \frac{3x+1}{x^2+3x-10}\, dx = \int \left(\frac{2}{x+5} + \frac{1}{x-2} \right) dx = 2\ln(x+5) + \ln(x-2) + C$$

4. Evaluate the following integral.

$$\int \frac{2x^2+5x+4}{x+1}\, dx = \int \left(2x+3+\frac{1}{x+1} \right) dx = x^2 + 3x + \ln(x+1) + C$$

5. Evaluate the following integral.

$$\int \frac{6x^2 - 5x + 3}{(x-1)(x^2+1)}\, dx = \int \left(\frac{2}{x-1} + \frac{4x-1}{x^2+1} \right) dx =$$

$$\int \left(\frac{2}{x-1} + \frac{4x}{x^2+1} - \frac{1}{x^2+1} \right) dx = 2\ln(x-1) + 2\ln(x^2+1) - \tan^{-1} x + C$$

6. Evaluate the following integral. For full points, you should simplify your final answer so that it does not include a trigonometric function applied to an inverse trigonometric function (i.e. $\sin\left(\cos^{-1} x\right)$, $\cos\left(2\tan^{-1} x\right)$, etc.).

$$\int \frac{1}{(x^2+1)^2}\, dx$$

Making the substitution $x = \tan\theta$ and then simplifying the integrand gives

$$\int \frac{1}{(x^2+1)^2}\, dx =$$

$$\int \cos^2\theta \, d\theta =$$

$$\int \left(\frac{1}{2} + \frac{1}{2}\cos(2\theta) \right) d\theta =$$

$$\frac{1}{2}\theta + \frac{1}{4}\sin(2\theta) + C =$$

$$\frac{1}{2}\tan^{-1} x + \frac{1}{4}\sin(2\tan^{-1} x) + C =$$

$$\frac{1}{2}\tan^{-1} x + \frac{x}{2(x^2+1)} + C$$