

1. An inverted conical tank has a 5 foot radius at the top and is 10 feet high. It is filled to within 2 feet of the top with olive oil weighing 57 lb/ft^3 . How much work does it take to pump the oil to the rim of the tank?

$$\begin{aligned}
 W &= \int_0^8 57\pi \left(\frac{1}{2}y\right)^2 (10 - y) dy \\
 &= \frac{57\pi}{4} \int_0^8 (10y^2 - y^3) dy \\
 &= \frac{57\pi}{4} \left[\frac{10}{3}y^3 - \frac{y^4}{4} \right]_0^8 \\
 &= \frac{57\pi}{4} [(5120/3 - 1024) - (0)] \\
 &= \frac{57\pi}{4} \cdot \frac{2048}{3} \\
 &= 9728\pi \\
 &\approx 30561.4 \text{ ft} \cdot \text{lb}
 \end{aligned}$$

2. A bathroom scale is compressed $1/16 \text{ in.}$ when a 150-pound person stands on it. Assuming the scale behaves like a spring,

- (a) How much does someone who compresses the scale $1/8 \text{ in.}$ weigh?

From Hooke's Law, we get $F(y) = -ky$. Substituting $y = -1/16$ and $F(y) = 150$ we get $k = 2400 \text{ lb/in.}$ Thus $F(y) = -2400y$ and we find that $F(-1/8) = 300 \text{ lb.}$

- (b) How much work does it take to compress the scale $1/8 \text{ in.}$

$$\int_{-1/8}^0 -2400y dy = 18.75 \text{ in} \cdot \text{lb}$$

3. Find the average value of the function $f(x) = 5 \sin\left(\frac{x}{2}\right)$ on the interval $[0, 2\pi]$.

$$\begin{aligned}
 f_{ave} &= \frac{1}{2\pi} \int_0^{2\pi} 5 \sin(x/2) dx \\
 &= \frac{5}{\pi} \int_0^{\pi} \sin u du \\
 &= \frac{-5}{\pi} [\cos u]_0^{\pi}
 \end{aligned}$$

$$= \frac{-5}{\pi} [(-1) - (1)]$$

$$= \frac{10}{\pi}$$

$$\approx 3.183$$