

Name _____

Determine if the following series converge or diverge. You must fully justify your answer.

1. (2 points) $\sum_{k=3}^{\infty} \frac{5^k}{k!}$

2. (2 points) $\sum_{k=1}^{\infty} \frac{1}{(3 + \ln k)^k}$

3. (2 points) $\sum_{k=5}^{\infty} \frac{1}{2 + \sqrt{k}}$

4. (4 points) The following infinite series converges to a sum S .

$$S = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{\sqrt{k}} = 1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{6}} + \dots$$

(a) There are two conditions that are necessary for an alternating series to converge. Explicitly show that both conditions are met here.

(b) If we approximate the sum of the infinite series with

$$S \approx \sum_{k=1}^3 (-1)^{k+1} \frac{1}{\sqrt{k}} = 1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}},$$

then how can the error in our approximation be bounded? (fill in the blank below)

$$|\text{error}| \leq$$

(c) How many of the beginning terms of the infinite series could you add together to get an estimate for its sum S that was within 0.01 of the correct sum?