

Test 1 Material

1. (10 points) Evaluate the following indefinite integrals.

(a) $\int \frac{1}{x} dx$

(b) $\int \sin x dx$

(c) $\int \cos x dx$

(d) $\int \frac{1}{\sqrt{1-x^2}} dx$

(e) $\int \sec^2 x dx$

(f) $\int \csc^2 x dx$

(g) $\int \frac{1}{1+x^2} dx$

(h) $\int \csc x \cot x dx$

(i) $\int \sec x \tan x dx$

(j) $\int e^x dx$

2. (15 points) Set up, but do not evaluate, one or more integrals which represent the area of the region between the graphs of $f(x) = x + 3$ and $g(x) = 9 - x^2$.

3. (15 points) Set up, but do not evaluate, one or more integrals which represent the volume obtained when the region bounded by $y = \ln x$, $y = 2$, $y = 0$, and $x = 0$ is revolved around the x -axis.

4. (20 points) Evaluate the following integral.

$$\int 6xe^{x^2} dx.$$

5. (20 points) Evaluate the following integral.

$$\int \frac{x}{x^2 + 4} dx$$

6. (20 points) Evaluate the following integral.

$$\int \frac{1}{x^2 + 4} dx.$$

Test 2 Material

7. (20 points) Evaluate the following integral.

$$\int x^2 \ln x \, dx$$

8. (20 points) Evaluate the following improper integral. (show each step clearly)

$$\int_1^{\infty} 6e^{-2x} \, dx$$

9. (20 points) Evaluate the following integral.

$$\int \frac{3x}{(x-2)(x+1)} dx$$

10. (15 points) Evaluate the following integral.

$$\int \cos^3 x \sin^2 x dx$$

11. (15 points) Evaluate the following integral.

$$\int x^2 e^x dx$$

12. (10 points) Evaluate the following integral.

$$\int \sin^2 x dx$$

Test 3 Material

13. (10 points) Find the Maclaurin series for $\cos(x^2)$. Include all terms up to x^8 .

14. (10 points) Find the interval of convergence for the following power series. You must thoroughly justify your claim.

$$\sum_{n=1}^{\infty} \frac{(2x - 1)^n}{n}$$

15. (5 points each) Compute exact sums for the following convergent series. No explanation is necessary.

$$(a) \sum_{k=0}^{\infty} \frac{(-1)^k}{2^{(k+1)}}$$

$$(b) \sum_{k=0}^{\infty} \left(\frac{1}{2k+1} - \frac{1}{2k+3} \right)$$

$$(c) \sum_{k=0}^{\infty} \frac{1}{k!}$$

16. (12 points each) Do the following series converge or diverge? Explain your reasoning.

(a)
$$\sum_{k=3}^{\infty} \frac{2k^2 + 1}{k^2}$$

(b)
$$\sum_{k=1}^{\infty} \frac{3}{2^k}$$

$$(c) \sum_{k=1}^{\infty} \frac{1}{1+k^2}$$

$$(d) \sum_{k=1}^{\infty} \frac{2k+3}{k^2+1}$$

$$(e) \sum_{k=2}^{\infty} \frac{k^2}{k!}$$

17. (5 points) Does the following series converge or diverge? Explain your reasoning.

$$\sum_{k=2}^{\infty} \ln \left(\frac{k}{k+1} \right)$$

Bonus material

18. (6 points) The polar equations for two curves are given below. Carefully sketch both of these curves and find their points of intersection. Each of the intersection points should be given both in polar coordinates and in cartesian coordinates.

- $r = 4$
- $r = 8 \cos \theta$