

Name _____

On a separate sheet of paper, answer all 5 parts to problem #1. The remaining problems are not to be turned in. They are problems that I've asked on quizzes and tests from previous semesters that I've taught this course.

1. Determine if the following series converge or diverge. Your entire grade will be based upon how clearly you write up and justify your solution. In particular you should specify which test (nth term, ratio, geometric, limit comparison, etc.) you are using, why it is applicable, and what the test implies about the convergence or divergence of the series.

(a)
$$\sum_{k=1}^{\infty} \frac{2^k}{k!}$$

(b)
$$\sum_{k=1}^{\infty} k \sin\left(\frac{1}{k}\right)$$

(c)
$$\frac{5}{3 \cdot 4} + \frac{5}{4 \cdot 5} + \frac{5}{5 \cdot 6} + \frac{5}{6 \cdot 7} + \frac{5}{7 \cdot 8} + \dots$$

(d)
$$\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$$

(e)
$$\sum_{k=1}^{\infty} \frac{10 + \sin k}{2^k}$$

2. Rewrite the following repeating decimals as infinite series and then compute their sums. See if you recognize a general pattern.

(a) 0.44444...

(b) 0.77777...

(c) 0.13131313...

(d) 0.05050505...

(e) 0.21111111...

3. Use three different tests to show that $\sum_{k=1}^{\infty} \frac{1}{1+k^2}$ converges.

4. Use the integral test to show that $\sum_{k=1}^{\infty} \frac{1}{k^3}$ converges. If you approximate the sum of this series

as $\sum_{k=1}^2 \frac{1}{k^3} = 1 + \frac{1}{8} = 1.125$, then what is the associated error term? How many terms could you add in the series if you wanted to guarantee that your approximation was within 0.005 of the correct sum?

5. Determine if the following series converge or diverge. You must fully justify your answer. If a series is a convergent geometric or collapsing series, then you should also compute its sum.

(a) $\sum_{k=1}^{\infty} \frac{3k^2 + 2k}{k^3 + k + 1}$

(b) $\sum_{k=1}^{\infty} \frac{\sqrt{k}}{k^3 + k}$

(c) $\frac{1}{2!} + \frac{4}{4!} + \frac{9}{6!} + \frac{16}{8!} + \frac{25}{10!} + \frac{36}{12!} + \frac{49}{14!} + \dots$

(d) $\sum_{k=1}^{\infty} \frac{k^3 + 1}{2k^5 - 1}$

(e) $8 - 4 + 2 - 1 + 1/2 - 1/4 + 1/8 - 1/16 + 1/32 - 1/64 + \dots$

(f) $\sum_{k=3}^{\infty} \left(\frac{1}{3}\right)^{k-1}$

(g) $\sum_{k=1}^{\infty} \frac{k}{e^{0.5k}}$

(h) $\sum_{k=0}^{\infty} e^{-k}$

(i) $\sum_{k=1}^{\infty} \frac{2^k - 1}{3^k}$

(j) $\sum_{k=1}^{\infty} \frac{2 + \sin k}{\sqrt{k}}$

(k) $\sum_{k=1}^{\infty} \left(\frac{9}{4}\right)^k$

(l) $\sum_{k=1}^{\infty} \frac{(-1)^k}{2^{2k+1}}$

(m) $\sum_{k=1}^{\infty} \frac{1}{2k}$

(n) $\sum_{k=1}^{\infty} \left(\frac{1}{(k+1)^2} - \frac{1}{(k+2)^2} \right)$

(o) $\sum_{k=1}^{\infty} \frac{2^{2k+1}}{3^k}$

(p) $\sum_{k=1}^{\infty} \left(\frac{k+1}{k} \right)^k$

6. Euler showed that

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \dots$$

Use this fact to find the sum of the series

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \frac{1}{9^2} + \frac{1}{11^2} + \dots$$