

Name _____

- No calculators are allowed.
- A correct answer will only be given full credit if enough work is shown to justify that answer.

1. (10 points) Prove that the following limit is correctly evaluated (use the $\epsilon - \delta$ definition of the limit).

$$\lim_{x \rightarrow 2} (5x - 3) = 7$$

2. (18 points) Find the exact value of the following limits.

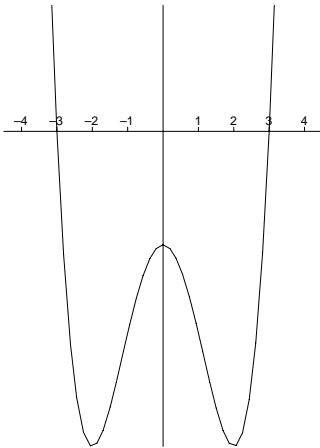
(a) $\lim_{x \rightarrow \infty} \frac{4x^2}{20 - 7x^2}$

(b) $\lim_{x \rightarrow 0} \frac{\sin(x)}{3x}$

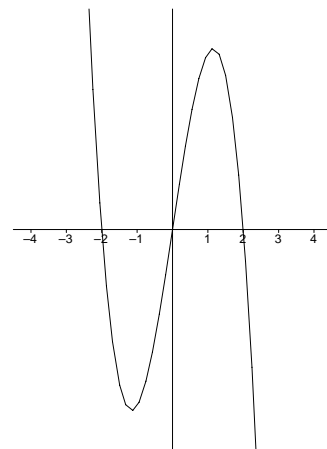
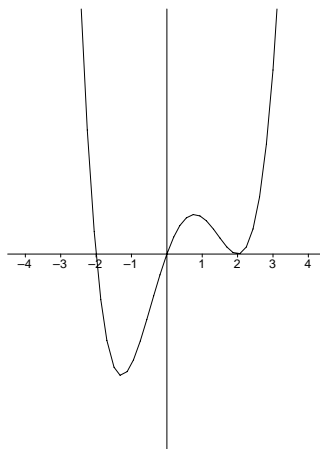
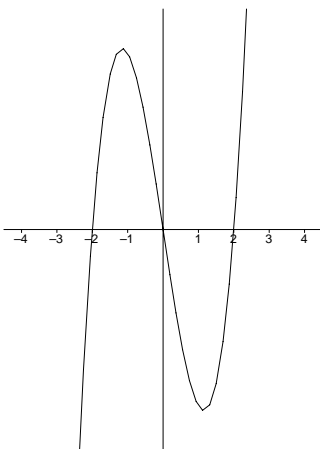
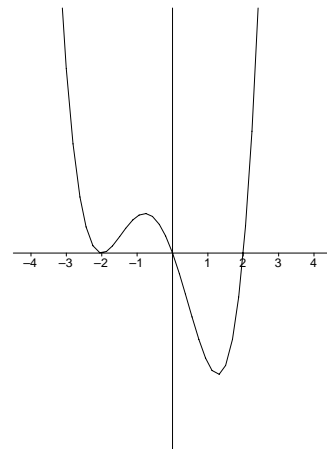
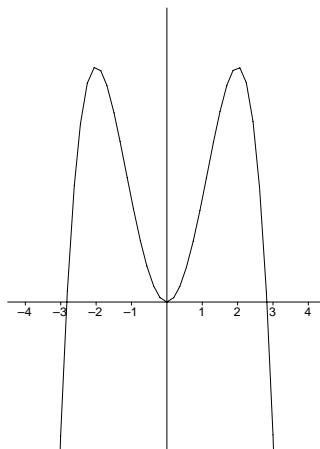
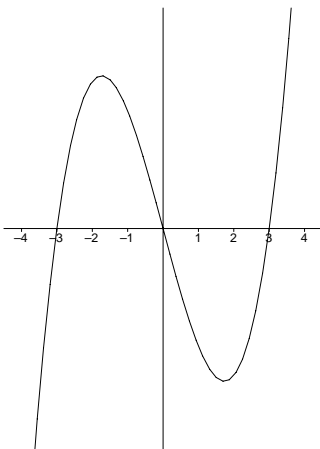
(c) $\lim_{x \rightarrow \infty} \frac{\sin(x)}{3x}$

3. (10 points) Let $f(x) = 3x^2 - 2$. Use the definition of a derivative (i.e. limits) to show that $f'(x) = 6x$. Show each step in your calculation and be sure to use proper terminology.

4. (10 points) The graph of $f(x)$ is shown below.



Circle the graph of $f'(x)$, given that it is one of the 6 choices below.



5. (42 points) Complete each boxed equation with the appropriate formula for the derivative. You do not need to simplify your answers.

(a) If $y = 5x^2 - 3x + 2$, then

$$\frac{dy}{dx} =$$

(b) If $y = \frac{1}{\sqrt{x}} + \frac{1}{3x^2}$, then

$$\frac{dy}{dx} =$$

(c) If $f(x) = x^2 \cos x$, then

$$f'(x) =$$

(d) If $g(x) = \frac{x^4 + x^2}{x^2 + 1}$, then

$$g'(x) =$$

(e) If $f(x) = \tan(x^2 + 1)$, then

$$f'(x) =$$

(f) If $f(x) = \sin^2(x^3)$, then

$$f'(x) =$$

6. (10 points) **Fact:** All polynomials of degree 3 will have either 1, 2, or 3 x -intercepts.

(a) Fill in the missing values from this table for the function $f(x) = 10x^3 - 20x - 5$.

x	$f(x)$
-3	-215
-2	-45
-1	
0	
1	
2	
3	205

(b) Based only upon your completed table of values, how many x -intercepts do you know to exist and where do they lie (i.e. between which successive pairs of integers)?

(c) Which theorem guarantees the existence of these x -intercepts, and which characteristic of polynomials is needed for this theorem to apply?