Math 122 (Section 9) Final Exam Review Sheet

Chapter 1

- Find the average rate of change of a given function. The function may be given as a table, as a graph, as a formula, or described in paragraph form (look at #1b,2b,7ab,8 in section 1.1, #12 in section 1.2, #5a,6b,10a in section 2.1).
- Determine if a function is linear, exponential, or neither. Write a formula for those that are linear or exponential. The function may be given as a table, or described in paragraph form (look at #7,8,25 in sectino 1.3, #1,3 in section 1.4, #10,11,12,13,14, 23 from section 1.5).
- Answer basic questions about cost and revenue. Mostly know what they measure, what fixed costs are, what marginal cost and marginal revenue measure, and how to calculate marginal cost and marginal revenue.

Section 2.1

- You will be given information about some function $P = f(t)$. Regardless of whether you are given a table of values, a graph, or a formula for P, you should be able to do the following:

  1. Compute total change in P between $t = a$ and $t = b$;
  2. Compute the average rate of change of P between $t = a$ and $t = b$;
  3. Compute the rate of change of P at some point $t = a$.
  4. From a graph, determine where the slope is positive, negative, or zero. Also determine where you have the greatest and least slopes.

  Be sure to include correct units for (1)—(3) above.

Section 2.2

- Given a function $y = f(x)$, $f'(a)$ denotes the derivative of f at the point $x = a$. All three of the following mean the exact same thing.

  - the derivative of f at a
  - the rate of change of f at a
  - the slope of the graph of f at a

  Since we've already dealt with slope and rate of change in 2.1, the problems in 2.2 are very similar. They simply use the new notation $f'(a)$. You are asked to calculate $f'(a)$ regardless of whether $f(x)$ is given as a table of values, a graph, or a formula. The graphical interpretation of total change, average rate of change, and rate of change at a point are also mentioned in 2.2. You may be given the graph of $f(x)$ along with two points $x = a$ and $x = b$. You should understand the graphical meaning of $f(b)$, $f(a)$, $b - a$, $f(b) - f(a)$, $\frac{f(b)-f(a)}{b-a}$, and $f'(a)$.

Section 2.3

- Given a function $y = f(x)$, we learned how to compute $f'(a)$ at any point $x = a$. So we see that $f'(x)$ is itself a function. Given a graph of $f(x)$, you should be able to sketch a graph of $f'(x)$. Remember that in the graph of $f'(x)$, the $y - values$ are just recording what the slopes are in the graph of $f(x)$. Another problem may give you information about $f'(x)$ and ask you to sketch a graph of $f(x)$. Knowing where f' is positive, negative, or zero tells you where f is increasing, decreasing, or constant - this enables you to sketch a graph of $f(x)$.

Section 2.4

- A new notation for the derivative is introduced. Given a function $y = f(x)$, $\frac{dy}{dx}|_{x=a}$ is equivalent to $f'(a)$. This new notation helps to remind us of the units for a derivative (i.e. the units for y divided by the units for x). In this section you are asked to give the practical meaning of statements such as $f'(8) = 4$. Knowing the units of f' makes this easier to do.

Section 2.5

- We are introduced to the second derivative. You will be given information about where f'' is positive, negative, or zero. Understand what this tells you about f' and what it tells you about f. You may be given partial information about f, f', or f'' and asked to either graph or answer questions about f, f', or f''.

Section 2.6

- Given a cost function and a revenue function, be able to compute marginal cost, marginal revenue, and profit. Know the connection between maximum profit and and these marginal quantities. Know the practical meaning of these marginal quantities.

Sections 3.1—3.2

- Given a table of values for the rate of change of some function, use Riemann sums to compute the total change of the function. Use your calculator to evaluate definite integrals.

Section 3.3 and 3.5

- Given a formula for some function, be able to compute a definite integral of that function on your calculator.

- Set up (and possibly evaluate) the definite integral (or integrals) needed to compute an area between two curves (one of these curves may just be the x-axis). (look at #3,4,11–15 in section 3.3)

- Given a graph of a function, be able to approximate (or find exactly if you see the area of basic shapes like triangles or rectangles) a definite integral of that function. Remember that the definite integral of a function gives the "signed area" between the graph of that function and the x-axis. (look at #1,2,6,19 in section 3.3)

- Given the rate at which some quantity is changing, be able to compute the total change in that quantity. (look at #5–9,14 in section 3.5)

Sections 5.1–5.4

- Given a formula for a function, be able to find a formula for the derivative of that function. In particular, if n, m, b, c, and a are constants ($a > 0$), then

  1. $\frac{d}{dx}\Big(x^n\Big) = nx^{n-1}$
  2. $\frac{d}{dx}\Big(\ln x\Big) = \frac{1}{x}$
  3. $\frac{d}{dx}\Big(a^x\Big) = \ln a \cdot a^x$
  4. $\frac{d}{dx}\Big(c\Big) = 0$
  5. $\frac{d}{dx}\Big(mx + b\Big) = m$
  6. $\frac{d}{dx}\Big(e^x\Big) = e^x$
  7. $\frac{d}{dx}\Big(f(x) \pm g(x)\Big) = f'(x) \pm g'(x)$
  8. $\frac{d}{dx}\Big(cf(x)\Big) = cf'(x)$
  9. **Chain Rule:** $\frac{d}{dx}\Big(f(g(x))\Big) = f'(g(x)) \cdot g'(x)$ $\quad\left(\text{also written as } \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}\right)$
  10. **Product Rule:** $\frac{d}{dx}\Big(f(x) \cdot g(x)\Big) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

  Note that rules 4 and 5 are really just special cases of rule 1. Also rule 6 is just a special case of rule 3 since $\ln e = 1$.

- Know what the derivative means (looking at chapter 2 and your old exams may help), and know how to apply this to problems like the following:

  1. Given a formula for some quantity, find the rate at which that quantity is changing.
  2. Sketch the graph of $f(x) =$ *some formula*. Find the slope of this curve at any given point. Also be able to sketch the tangent line at any particular point and find the equation of that tangent line.
  3. The cost (or revenue) function for some item is given by *some formula*. Find a formula for the marginal cost (or revenue). For a particular value of q, explain what this means in practical terms.
  4. The position of an object is given by *some formula*. Find a formula for the velocity of that object. Use this to answer questions like #43 in section 5.1.

Section 5.7

- Find indefinite integrals

- Evaluate definite integrals using the Fundamental Theorem of Calculus.