

Name _____

Seat # _____

- Do not open this test booklet until told to do so.
- Turn off all cell phones.
- For multiple-choice questions, precisely one answer is correct. Circle this correct answer.
- For all other questions, you must show sufficient work to justify your answer.
- **No calculators allowed!**
- Show your Student ID when you turn in your test.

Do not write below this line

#1 (5 points) _____

#11 (5 points) _____

#2 (5 points) _____

#12 (5 points) _____

#3 (5 points) _____

#13 (5 points) _____

#4 (5 points) _____

#14 (5 points) _____

#5 (5 points) _____

#15 (5 points) _____

#6 (5 points) _____

#16 (5 points) _____

#7 (5 points) _____

#17 (5 points) _____

#8 (5 points) _____

#18 (5 points) _____

#9 (5 points) _____

#19 (5 points) _____

#10 (5 points) _____

#20 (5 points) _____

Total (100 points) _____

1. (5 points) [simplify your answer] If $g(t) = 10t^5 - 5t^2 + 6$, then

$$g'(t) =$$

2. (5 points) If $h(x) = 3e^x - 5^x$, then

$$h'(x) =$$

3. (5 points) If $y = 2 + 5 \ln t$, then

$$\frac{dy}{dt} =$$

4. (5 points) [simplify your answer] If $y = \frac{-5}{2x^2}$, then

$$\frac{dy}{dx} =$$

5. (5 points) If $y = \sqrt[3]{x}$, then

$$\frac{dy}{dx} =$$

6. (5 points) If $w = e^{8t}$, then

$$\frac{dw}{dt} =$$

7. (5 points) [simplify your answer] If $P(t) = 5t^3 \ln t$, then

$$P'(t) =$$

8. (5 points) If $y = \frac{t^4}{t^2 + 5}$, then

$$\frac{dy}{dt} =$$

9. (5 points) [simplify your answer] If $y = (e^{5\ln x})^2$, then

$\frac{dy}{dx} =$

10. (5 points) Ralph has a man-made pond in his backyard. The pond had no fish so he purchased some guppies. They reproduced many times and Ralph noted that the total number of guppies could be approximated by the function $g(t) = t^2 + 50$, where t represents the number of months since his original purchase. Precisely ten months after his original purchase, the total number of guppies in his pond are increasing by
- (a) 15 guppies per month
 - (b) 20 guppies per month
 - (c) 50 guppies per month
 - (d) 70 guppies per month
 - (e) 100 guppies per month
 - (f) 150 guppies per month

11. (5 points) If $f(x) = \ln(3x^2)$, then what is the value of $f'(2)$?

(a) $\ln(2)$

(b) $\frac{1}{\ln(2)}$

(c) $\ln(12)$

(d) $\frac{1}{\ln(12)}$

(e) $\frac{1}{2}$

(f) $\frac{1}{3}$

(g) 0

(h) 1

(i) e^{12}

(j) $\frac{1}{e^{12}}$

12. (5 points) On the graph of $g(x) = 5x^2 + 100$, what is the slope of the curve at $x = 4$?

(a) 4

(b) 5

(c) 10

(d) 25

(e) 40

(f) 50

(g) 100

(h) 140

(i) 200

13. (5 points) Find the equation of the line tangent to the graph of $f(x) = x^2 + 3x + 2$ at $x = 1$.

(a) $y = 2x + 1$

(b) $y = 2x + 2$

(c) $y = 2x + 3$

(d) $y = 2x + 6$

(e) $y = 3x + 1$

(f) $y = 3x + 2$

(g) $y = 3x + 3$

(h) $y = 3x + 6$

(i) $y = 5x + 1$

(j) $y = 5x + 2$

(k) $y = 5x + 3$

(l) $y = 5x + 6$

14. (5 points) Below is a graph of the function $g(x) = \frac{\ln x}{x}$. What are the coordinates (x, y) for the local maximum value shown? Be sure each coordinate is in simplified form.

15. (5 points) A function $f(x)$ is given below along with its first and second derivatives in factored and unfactored forms.

- $f(x) = x^4 - 4x^3 + 16x - 16 = (x + 2)(x - 2)^3$
- $f'(x) = 4x^3 - 12x^2 + 16 = 4(x + 1)(x - 2)^2$
- $f''(x) = 12x^2 - 24x = 12x(x - 2)$

The graph of $f(x)$ is concave down upon which one of the following intervals?

- (a) $(-2, 2)$
- (b) $(-1, 2)$
- (c) $(0, 2)$
- (d) $(-\infty, -2)$
- (e) $(-\infty, -1)$
- (f) $(-\infty, 0)$
- (g) $(-2, \infty)$
- (h) $(-1, \infty)$
- (i) $(0, \infty)$
- (j) $(-\infty, \infty)$

16. (5 points) A function $g(x)$ has the following derivative.

$$g'(x) = 5e^x(x - 1)^2(x - 2)^3(x - 3)^4$$

Which one of the following statements is true about the graph of $g(x)$?

- (a) There is a local minimum at $x = -1$
- (b) There is a local minimum at $x = 0$
- (c) There is a local minimum at $x = 1$
- (d) There is a local minimum at $x = 2$
- (e) There is a local minimum at $x = 3$
- (f) There is a local maximum at $x = -1$
- (g) There is a local maximum at $x = 0$
- (h) There is a local maximum at $x = 1$
- (i) There is a local maximum at $x = 2$
- (j) There is a local maximum at $x = 3$

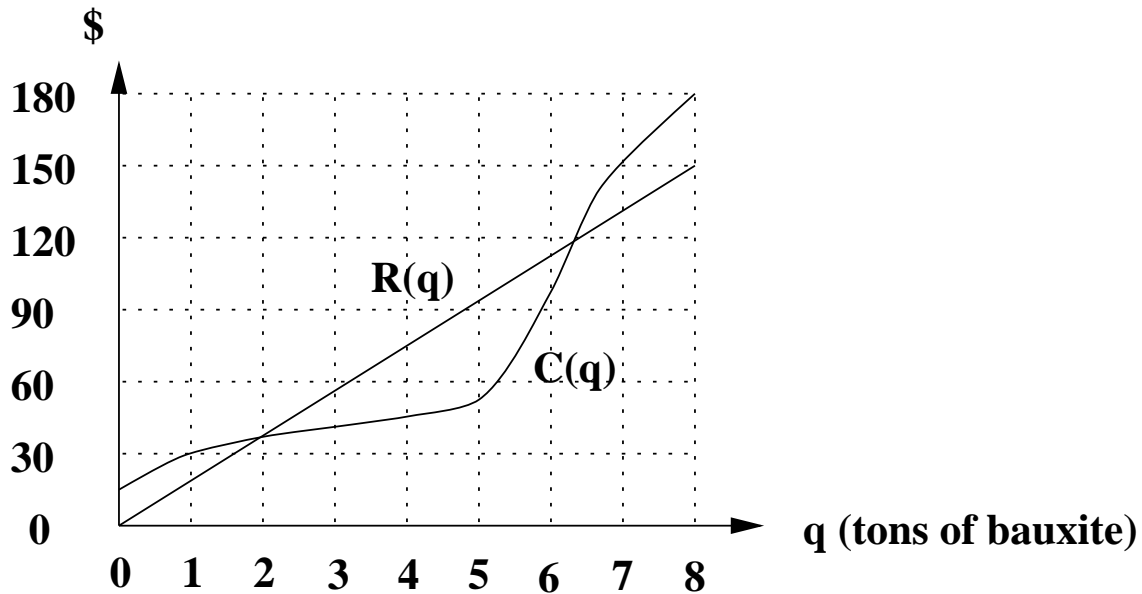
17. (5 points) A ball is tossed straight up with an initial velocity of 16 feet per second. The ball is 5 feet above the ground when it is released. Its height at time t is given by

$$h = -16t^2 + 16t + 5$$

How high does it go before returning to the ground?

- (a) 5 feet
 - (b) 6 feet
 - (c) 8 feet
 - (d) 9 feet
 - (e) 12 feet
 - (f) 16 feet
 - (g) 24 feet
 - (h) 32 feet
18. (5 points) To produce 20 items, the total cost is \$600 and the marginal cost is \$10 per item. Estimate the cost of producing 23 items.
- (a) \$230
 - (b) \$600
 - (c) \$610
 - (d) \$623
 - (e) \$630
 - (f) \$690
 - (g) \$720
 - (h) \$830

19. (5 points) A company in Brazil mines bauxite which it sells to other companies who use the bauxite to produce aluminum. Suppose that $C(q)$ represents the cost in dollars for this company to mine q tons of bauxite, and that $R(q)$ represents the revenue the company will receive when it sells q tons of bauxite. The graphs of $C(q)$ and $R(q)$ are shown below.



- (a) What is the exact price that the company charges for each ton of bauxite sold?
- (b) At a production level of 7 tons of bauxite, which is greater — the marginal revenue or the marginal cost?
- (c) At what production level does marginal revenue equal marginal cost?
- (d) Determine the number of tons of bauxite that the company must produce and sell in order to maximize profit. What is the dollar amount of that maximum profit?

20. (5 points) Rudy wants to enclose a 100 square foot rectangular region to be used for a garden. He will use fencing which costs \$16 per foot along three sides, and fencing which costs \$34 per foot along the fourth side. Let x represent the length of that fourth side.

(a) Find a formula for the total cost of all the fencing as a function of x .

(b) Find the dimensions which give the minimum total cost.

(c) Find the minimum total cost.