**BIOL 763** **Final Exam** **Name:**____________________

**Spring, 1999**

This is a take-home exam, due at our final session: Saturday, May 1, 2pm. You may use your notes, texts, or papers, but you must NOT collaborate. Be sure to supply adequate explanation for your answers. In many of the problems the later parts do not depend heavily on the earlier parts, so don't give up on (c) if you didn't get (a) or (b)! There are 150 points total.

1. (10 points) Considering that chaos is discussed in the context of deterministic systems, in what sense is chaos chaotic? Given data, how can one try to distinguish chaos from random noise?

2. (30 points) Consider these phase plots for three continous models.

   a. For each system, discuss the stability of the equilibrium solution (heavy dot).

b. Suppose in each case there is an initial condition of $x = 1.2$, $y = 1$. Descibe what happens as $t \to \infty$. How sensitive is this behavior to a small change in the initial condition?

c. Suppose the eigenvalues for the linearized system at the equilibrium are $\lambda_1 = a + bi$ and $\lambda_2 = c + di$. For each system, describe $a$, $b$, $c$, and $d$ in terms of being positive, negative, zero, non-zero, or simply as not determined by the graph.

3. (10 points) Discuss the relationship between the bifurcation plot and each of the plots A–F of population as a function of time. What does $K$ on the bifurcation plot represent in terms of the plots of $N(t)$ versus $t$? For each plot, supply the appropriate value (or range of values) of $r$ to complete the caption "when $r$ is . . . "

4. (30 points) The Leslie matrix for a population of New Zealand sheep is given by the following data, where $F_i$ denotes the fecundity and $P_i$ the probability of survival to the next age class.

| age (yr) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $F_i$ | .0 | .04 | .39 | .47 | .48 | .55 | .54 | .50 | .47 | .46 | .43 | .42 |
| $P_i$ | .84 | .98 | .96 | .95 | .93 | .90 | .85 | .79 | .69 | .56 | .37 | .0 |

a. Write out the upper left $4 \times 4$ corner of the matrix $L$.

b. If the current population consists of 100 two year olds and 100 three year olds how many individuals will be in each age class one year from now?

c. The dominant eigenvalue for $L$ is $\lambda = 1.18$, and the corresponding eigenvector is $[.24, .17, .14, .12, .10, .08, .06, .04, .03, .02, .01, .0]$. The dominant eigenvalue for $L^T$ is $\mu = 1.18$, and the corresponding eigenvector is $[1.0, 1.4, 1.6, 1.6, 1.5, 1.3, 1.1, 0.9, 0.8, 0.6, 0.5, 0.4]$. Over the long term does the population grow, decline, or remain stable? Why? In the long term what percent of the population is 7 years old or older? Which age class has the highest reproductive value? Is this the age class with the highest fecundity? If not, explain the apparent discrepancy.

d. Recall that the elasticity $e_{ij}$ times percent change in $L_{ij}$ gives percent change in $\lambda$. We have calculated that $e_{13} = 0.037$ and $e_{18} = 0.014$.

Suppose the fecundity of the two year olds rises from .39 to .44 and the fecundity of the seven year olds drops from .50 to .40. Which has a larger effect on $\lambda$ in terms of percent change? How do you account for this intuitively?

5. (20 points)

a. You are given the plot $N_{t+1} = F(N_t)$. Based on this graph, what is the steady state $\bar{N}$? If $N_0 = 5$, use the graph to compute $N_4$.

b. You are given the plots $N_{t+1} = G(N_t)$, $N_{t+2} = G(N_{t+1}) = G(G(N_t))$, and the line $N_{t+2} = N_t$. Interpret the points A, B, and C in terms of stable or unstable equilibria and cycles. If $N_0 = 1.1$, what is the long term behavior of $N_t$?

6. (30 points) In this problem we investigate a host-parasitoid interaction in which the hosts have a possible refuge. Let $N_t$ denote the host population at time $t$ and $P_t$ the parasitoid population. In the absence of parasitoids the host population grows at a per capita rate $\lambda$. Each parasitized host yields $c$ parasitoids in the next generation. The probability of $k$ episodes of parasitoid attack on an unprotected host in one breeding cycle is $p(k) = e^{-aP_t}(aP_t)^k/k!$. The carrying capacity of the host population is $K$ and a fraction $b$ of this amount can hide in a protected refuge. All parameter values are positive.

   a. How many of the host population can hide? What fraction of the total host population is this? What fraction of the host population does this leave exposed?

   b. What is the probability that an exposed host escapes being parasitized (or, what fraction of the exposed host population escapes being parasitized)?

c. What fraction of $N_t$ is available for reproduction, either by hiding, or by being exposed but lucky?

d. Explain how the equations given below reflect our description of the model.

$$\begin{aligned} N_{t+1} &= \lambda N_t \left( \frac{bK}{N_t} + \left(1 - \frac{bK}{N_t}\right) e^{-aP_t} \right) \\ P_{t+1} &= cN_t \left(1 - \frac{bK}{N_t}\right) (1 - e^{-aP_t}) \end{aligned}$$

e. There is a potential flaw in this model. What does it predict for very low host populations (for example, around $\frac{1}{2}bK$ )?

f. With the parameter values $a = 0.2$, $b = 0.1$, $c = 1$, $K = 100$, $\lambda = 2.8$, we find that there is a steady state $\bar{N} = 29.25$, $\bar{P} = 18.81$;

the corresponding Jacobian matrix has eigenvalues $0.08 \pm 0.49i$ . Describe the behavior of the system near the equilibrium.

7. (20 points) Consider a population in which there are two types of individuals $S$ and $T$ which may come into conflict. The fitness of an individual is its baseline fitness $W_0$ plus the fitness change resulting from an encounter with another individual weighted by the probability of such an encounter.

$$\begin{aligned} W(S) &= \quad W_0 + pE_{SS} + (1-p)E_{ST} \\ W(T) &= \quad W_0 + pE_{TS} + (1-p)E_{TT} \end{aligned}$$

At time $t$ the proportion of $S$ type individuals is $p_t$ , and of $T$ type individuals $1-p_t$ . The proportion of type $S$ individuals in the next generation is dependent upon the proportion in the current generation and the ratio of fitness of type $S$ individuals to average fitness. We have the following fitness changes (payoffs): $E_{SS} = 2$ , $E_{ST} = 0.5$ , $E_{TS} = -0.5$ , $E_{TT} = 1$ , where $E_{AB}$ denotes the fitness change for $A$ in an encounter with $B$ (taking $A$ to be the invader of the territory occupied by $B$ ). Under these conditions the system is in equilibrium if $p = 0.5$ . If, however, at time $t = 0$ we have $p_0 = 0.4$ , then $p_5 = 0.32$ , $p_{10} = 0.21$ , and $p_{20} = 0.03$ ; on the other hand, if $p_0 = 0.6$ , then $p_5 = 0.67$ , $p_{10} = 0.78$ , and $p_{20} = 0.96$ .

a. What gain or loss in fitness does type $S$ have if $p_0 = 0.6$ ? What gain or loss in fitness does type $T$ have under the same initial condition?

b. Is the mixed equilibrium strategy (50% $S$ and 50% $T$) stable to invasion by a different strategy? Explain. Is this equilibrium an ESS?

c. What are the ESS strategies, if any, in this population? Give some intuitive justification for your answer.