BIOL 763 / MATH 599 / SCCC 411B
Final Exam   **Fall, 2003**   **Name:**_____

Instructions. This is a take-home exam. You may use class notes, the text or other books, and you may ask us about specific problems with Maple code. Other than that you may not consult with anyone. There are 150 points. Answer all questions. **Be sure to supply adequate explanation for your answers**. You may attach additional sheets; if you do, staple them at the end, using a fresh sheet for differently numbered problems, and indicate in the problem space that you are doing this! This exam is due 9 am sharp on Tuesday, December 9, one copy to each of our offices or mailboxes.

1. (20 points) Give brief definitions of the following terms. Illustrative graphs may also be appropriate.
   a.   equilibrium

   b.   nullcline

   c.   eigenvalue–eigenvector pair

   d.   Jacobian matrix

   e.   difference equation

2. (15 points) The Leslie-Lefkowitch matrix for a certain population model with

classes I, II, III, and IV is $A = \begin{bmatrix} 0 & 0.5 & 4.0 & 0 \\ 0.9 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.3 & 0.2 \end{bmatrix}$

a. Does this model describe age classes or stages? Why? (Suggestion: what is the significance of the last entry in the bottom row?) Which age classes/stages are active reproductives? Explain.

b. If the current population consists of 100 individuals in class II, how many individuals will be in each class two time steps from now?

c. The dominant eigenvalue for this system is $\lambda = 1.53$, and a corresponding eigenvector is $[0.51, 0.3, 0.16, 0.04]$. Over the long term does the population grow, decline, or remain stable? What is the long term distribution of the population in percent terms?

3. (15 points) You are given the plot $N_{t+1} = F(N_t)$. Based on this graph, what is the non-zero steady state $\bar{N}$? If $N_0 = 2$, use the graph to compute N_4. Discuss the stability properties of the equilibria (stable / unstable), and, in addition, the local behavior of the system near the equilibria in terms of the slope of the appropriate tangent line (this can be computed by drawing the tangent line, picking off points on the line, and computing the slope, or since the scales are the same on the two axes, simply by using a ruler).

4. (10 points) Copy your favorite bifurcation plot (give the source) and attach it. Show where stable 4-cycles appear on the plot. Choose one particular 4-cycle, sketch N_t as a function of t, and discuss how the shape and values of this plot are related to the values on the bifurcation plot.

5. (10 points) Do problem 4 on page 277 of Case. Assume that the prey grow logistically by $dN/dt = rN(1 - N/K) = rN - rN^2/K$, so the slope at $N = 0$ is r. The peak growth rate occurs when $N = K/2$. A type III functional response

is given by $F(N) = HN^2/(A^2 + N^2)$, and $F'(0) = 0$. Use a combination of graphical and analytical arguments to support your answer.

6. (15 points) Consider the four phase diagrams below for a continuous sytem.

   a. Suppose the eigenvalues for the linearized system at the equilibrium (marked by the heavy dot) are $\lambda_1 = a + bi$ and $\lambda_2 = c + di$. For

each system, describe a, b, c, and d in terms of being positive, negative, zero, or simply non-zero, as is most appropriate.

b. In which system(s) can a small change in the initial condition lead to a large difference in the long term trajectories? Exhibit this on the plot(s).
c. Assuming no radical change in behavior, which, if any, of the systems might exhibit a limit cycle? Sketch what it would look like.

7. (40 points) In this problem we investigate a vegetation-herbivore interaction given by

$$\frac{dV}{dt} = aV\left(1 - \frac{V}{K}\right) - \frac{bV}{e+V}H$$
$$\frac{dH}{dt} = \frac{cV}{e+V}H - dH$$

Here V is measure in units of mass, say, Kg, and H is a population count. All parameter values are positive. In parts (a), (b), and (c) it may be helpful to relate this model to standard models.

a. What happens to the vegetation in the absence of the herbivores? What happens to the herbivores in the absence of vegetation?

b. If V is small in comparison to e, what can you conclude about the growth term of the herbivore population?

c. If V is large compared to e, what can you conclude about the growth term of the herbivore population?

e. There is a non-trivial steady state $(\bar{V}, \bar{H})$. Calculate just $\bar{V}$ (suggestion: use the second equation to solve for $\bar{V}$ in terms of the parameters).

d. What further restriction must be imposed on the parameter values to be sure the steady state is actually feasible?

e. We have sketched the nullclines of the system using parameter values $a = 0.5$, $b = 0.001$, $c = 0.5$, $d = 0.4$, $K = 500$, and $e = 100$. The equilibrium values are $\bar{V} = 400$ and $\bar{H} = 50,000$. Determine which nullcline corresponds to $\frac{dV}{dt} = 0$ and which to $\frac{dH}{dt} = 0$. Then determine

what happens to H if you start a trajectory in each of the four regions; do the same for V. Use arrows to indicate the direction of net change.

f.   At the equilibrium the Jacobian matrix has eigenvalues $\lambda_1 = -0.027$ and $\lambda_2 = -0.292$. Is the equilibrium stable or not? Sketch a plausible trajectory on the graph above, if the initial state is $V = 450$ and very small H (a new infestation). What do you expect to happen in the long term?

8. (10 points) Considering that chaos is discussed in the context of deterministic systems, in what sense is chaos chaotic?

9. (15 points) Do problem 5 from page 310 of Case. Answer the main part of the
   question, and parts a, b, c, and d. Very briefly justify your true / false answers.