BIOL 763 / SCCC 411B **Name:**______________________

Final Exam **Fall, 2001**

This exam is due on Friday, December 14. You are to work individually, but you may use your texts, notes, or Maple. Give sufficient justification for your answers. (Don't sweat the Maple–if you know what you want it to do, but don't see how to get the existing worksheets to do it, please ask one of us for help!) In many of the problems, the later parts do not depend heavily on earlier parts, so don't give up on (c) if you didn't get (a) or (b)! Please use your own paper, number the pages, and clearly number the problems and the parts. (Use the exam paper, or a tracing of it for the graphical problems.) There are 200 points.

1. (25 points) Briefly discuss each of the following terms. Illustrative pictures, formulas, or applications might also be appropriate to amplify the definition.

   a. type 2 functional response

   b. eigenvalue-eigenvector pair

   c. non-linear differential equation

   d. Jacobian matrix

   e. Leslie matrix

2. (20 points) A population $N(t)$ of turtles has a *per capita* growth rate given by

$$(0.2)(1 - (5/N) - (N/40))$$

   in units of (turtles/year)/turtle.

   a. Write the equation for the *net* growth rate $\frac{dN}{dt}$. What are the units?

   b. This system is found to have two non-trivial equilibrium values: $N = 5.9$ and $N = 34.1$. Analyze the stability of these equilibria by selecting initial values close to them and using the differential equation to decide whether there should be growth or decline at those points. Sketch N as a function of t for each case (all on one graph) as $t \to \infty$.

   c. This kind of model is often used in a context in which it is believed that there is a "minimal sustainable population." How can this be seen in terms of the results you obtained above?

3. (30 points) Consider these phase plots for three continous models.

a. For each system, discuss the stability of the equilibrium solution (heavy dot).

b. Suppose in each case there is an initial condition of $x = 0.8$, $y = 0.9$. Descibe what happens as $t \to \infty$. How sensitive is this behavior to a small change in the initial condition?

c. Suppose the eigenvalues for the linearized system at the equilibrium are $\lambda_1 = a + bi$ and $\lambda_2 = c + di$. For each system, describe a, b, c, and d in terms of being positive, negative, zero, non-zero, or simply as not determined by the graph.

4. (20 points) Hitchcock and Gratto-Trevor (1997, Ecology 78: 522-534) discuss the dynamics of a population of semipalmated sandpipers in Manitoba. They report the following population projection matrix.

$$A = \begin{bmatrix} 0.021 & 0.074 & 0.085 \\ 0.563 & 0 & 0 \\ 0 & 0.563 & 0.563 \end{bmatrix}$$

The dominant eigenvalue is $\lambda = 0.639$ and the stable age distribution vector is $[0.119, 0.105, 0.776]$.

a. What does the model predict about the long term for this population?

b. The authors propose the following model for this population: $N_{t+1} = AN_t + I_t$, where I is a vector of immigrant birds not hatched at the site. In 1985 the authors observed 80 breeding pairs. Assuming the population was at its stable age distribution, how many breeding pairs should have been observed in 1986? In fact 57 breeding pairs were observed, including 5 immigrant pairs. On the basis of this information, do you think the model is worth further investigation?

c. Can you identify a possible flaw in the model if you observe that the immigrant pairs are all fully mature adults?

5. (10 points) You are given the plot $N_{t+1} = F(N_t)$. Based on this graph, what is the steady state $\bar{N}$? If $N_0 = 5$, use the graph to compute N_4.

6. (10 points) Do problem 4 on page 277 of the text.

7. (15 points) Use the diagram from problem 11 on page 342 of the text. If species 1 is at its carrying capacity and species 2 is just beginning to invade (so has a very small, but positive, population), what is the predicted outcome? Can you suggest a biological reason why the nullclines here are not the usual straight lines of a classical Lotka-Volterra competition model?

8. (10 points) Considering that chaos is discussed in the context of deterministic systems, in what sense is chaos chaotic? Given data, how can one try to distinguish chaos from random noise?

9. (60 points) In a predator-prey system developed by Robert May, with suitable units for prey x and predators y, we have:

$$\begin{aligned}\frac{dx}{dt} &= 0.6x(1-\frac{x}{5}) - 0.5\frac{xy}{x+1}\\ \frac{dy}{dt} &= 0.1y(1-\frac{y}{2x})\end{aligned}$$

a. In the absence of predators, what does this model predict?

b. If prey are extremely abundant, what does this model predict about the predator population in the near term?

c. Is it possible for the predator population to decline in this model? Under what circumstances?

d. Identify the term that gives the loss in prey population due to predation. Explain the significance of this term for very low levels of prey.

e. Nullclines for this system are plotted below. Indicate which one goes with $\frac{dx}{dt} = 0$ and which one with $\frac{dy}{dt} = 0$. In each of the four regions cut out by the nullclines decide whether each of x and y must be increasing or decreasing. Give an arrow that represents the net direction a trajectory must have as it passes through each region. (Suggestion: you may find it useful to decide what happens to x if an initial condition lies under the arched curve, and what happens to y if an initial condition lies above and to the left of the straight line.)

f. When Maple computes the steady states, it finds one at (-5.3,-10.6). Why did we not include this one on the graph?

g. There is another steady state at (0.95,1.89). If the system is linearized with this point as the coordinate center, one finds a coefficient matrix with eigenvalues $\lambda = 0.01 \pm 0.19i$. Based on this information, and possibly also your answer to part (e), sketch a likely trajectory on the graph if the system begins with initial condition $x = 1$, $y = 2$.