

MATH 700
Fall, 2008

Quiz 1

Name: _____

This quiz is for real.

1. Suppose that a vector space V is a sum of subspaces W_i over the index set $I = \{1, \dots, n\}$. Show that the sum is direct if for each $j \in I$,

$$W_j \cap \text{span}\left(\bigcup_{\substack{i \in I \\ i \neq j}} W_i\right) = \{0\}.$$

2. Suppose V and W are vector spaces and $T \in L(V, W)$. Suppose that T is surjective (onto). Prove that there exists an $S \in L(W, V)$ such that $T \circ S = \text{id}_W$. Bonus: Prove that $V = N(T) \oplus R(S)$ (internal direct sum).