MATH 700 **Fall, 2008**

Linear Algebra

Instructor. Matt Miller, office LeConte 300I (also check LC 411), hours: MW 2:00-4:00, TTh 10:30-12:30, whenever you can find me in (which is often), and by appointment, phone: 777-3690, e-mail: `miller@math.sc.edu`, web: `http://` `www.math.sc.edu/` ∼ `miller/700`. Notice that these are ordinary email and website addresses, NOT accessed through Blackboard.

Text. Required: *Linear Algebra Done Right* by Sheldon Axler, Springer, 2nd. ed., 1997.

Prerequisite. You should have already completed an undergraduate course in linear algebra or matrix theory, be familiar with Gauss-Jordan row reduction for solving systems of linear equations and finding matrix inverses, know how to compute determinants, and be familiar with the vector spaces $\mathbf{R}^2$ and $\mathbf{R}^3$.

Overview. You may find that this course is very different from the corresponding undergraduate course. We will be less concerned with the mechanical aspects of computation, and more concerned with why we want to do these calculations, and the theory that underlies them. We will not restrict ourselves to finite dimensional spaces, but where we can, we will consider the extent to which results hold true in the infinite dimensional case as well. We will carefully define terms, and prove almost all results, sometimes in class, and sometimes in the homework. To accomplish this, solutions must be communicated effectively, both in writing and orally, and you will get practice doing this.

Learning objectives. Students will master concepts and solve problems based upon vector spaces, linear transformations, dual and inner product spaces, decompositions of spaces, and canonical forms.

Course content. We will follow the book rather closely, but with some additions and subtractions. Contra Axler, for us polynomials are formal objects and not functions, and I will assume that you at least recognize the results of Chapter 4 (and believe that they hold, with obvious exceptions such as the fundamental theorem of algebra, over arbitrary fields–in any case you'll see them in full in MATH 701). We will conclude chapter 7 with the section on normal operators, but we will add a broader discussion of duality not restricted to inner product spaces. We will replace chapter 9 by using complexification of real vector spaces to obtain the Cayley-Hamilton Theorem for real operators, proceed to chapter 10, then revisit minimal and characteristic polynomials, and finally extract criteria for diagonalization over arbitrary fields. Time permitting (but it probably won't) we'll look at the rational canonical form. Besides greatly re-ordering the topics of the course (with which I am in pretty good agreement), Axler introduces several smaller novelties in definitions and notation. I favor some, am neutral about others, and don't really object to any; but i will inform you of the more traditional language because you will see it later.

Grades. Two major tests will be given, each worth 100 points, on Thursday, September 25 (day 11) and Thursday, November 20 (day 25). There will be a couple of quizzes at the beginning on the prerequisite material; these are for your information only. There will be around five short quizzes to be sure you are keeping up with the material (and attending

class); I will count the best three. **No make-ups will be given for exams or quizzes**, but part of your score on your final will replace your lowest exam score. There will be a comprehensive final exam on Thursday, December 11 at 5:30 pm., worth 150 points. **No exemptions will be granted**. Though comprehensive, it will emphasize the latter part of the course more heavily, as this is the part that is generally emphasized on the Admission to Candidacy (Qualifying) Exam. Homework will be assigned regularly and selected problems will be graded. Each problem will be graded on the scale: 4 perfect; 3 minor error, gap, or "wandering around"; 2 significant progress, but major error or gap; 1 just a start in the right direction; 0 insufficient, wrong, or incoherent. *Notice that a mathematically correct solution may not receive full credit if the writing is not "clean": you should say everything that has to be said, but no more, and in the correct order.* Some of you have had some of this material before and will be tempted to write "obviously" or "clearly", etc. Resist this temptation; in this class *all* significant theoretical details must be shown (it is not necessary to show arithmetic details). When in doubt, ask! A total of 500 points may be earned:

Exams	200	
Final	150	
Quizzes	30	(best three)
Homework	120	(scaled score)

Letter grades will be announced separately for each exam, for the final, and for the overall homework and quiz totals. They will generally fall close to the scale 85–100 A, 75–84 B, 65–74 C, 55–64 D, below 55 F, but will vary up or down. Note that the deadline to drop this course without a grade of WF is Thursday, September 29; you should have a pretty good idea before then how you are doing.

Collaboration. One of the goals of this course is to learn how to communicate mathematical ideas. You will be expected to work with one another on the homework! We will have an optional, informal, student run, problem session, tentatively on Fridays at 3:00 (concluding in time for the traditional Math Department graduate student "Happy Hour" *aka* "Fluid Dynamics Seminar") that I will attend as a sort of coach. However, you will have to take the exams individually, so don't get too dependent upon one another. You will find that many of the exams in the graduate program are not proctored. According to the USC Student Handbook code of student academic responsibility, "**the first law of academic life is intellectual honesty.**" I expect this of all of you. If you are ever in the least bit uncertain about the ground-rules, ask for clarification!

Attendance. Regular attendance is important for success in this course. I intend that this be a very rich and varied class, and not all topics will be in the text. I'll be looking for students to present problems for token bonus "board points." If you do miss a class, you can find homework and a very brief synopsis on the class home page `http://www.math.sc.edu/ ~ miller/700`.