MATH 700 **Quiz #2** **Name:**______________________
Fall, 2008

1. Let n be a positve natural number, A be an $n \times n$ matrix over a field F, and $T \in L(F^n)$. In each case determine if A or T, respectively, is invertible, not invertible, or there is not sufficient information to decide. Justify your answer.
   a. $T^k = 0$ for some $k \geq 2$.

   b. $AB = 0$ for some nonzero $n \times p$ matrix B with $p \geq 1$.

   c. A is similar to an invertible $n \times n$ matrix B.

   d. $\text{nullity}(T) > \text{rank}(T)$.

2. Suppose V is an n-dimensional vector space, $n > 0$, and $T \in L(V)$. Let $\mathbf{v}$ be a non-zero vector in V. Explain why $\alpha = \langle \mathbf{v}, T\mathbf{v}, T^2\mathbf{v}, \ldots, T^n\mathbf{v} \rangle$ must be dependent, and why $\text{span}(\alpha)$ must be T-invariant.