If C_1 and C_2 are oriented curves with C_1 ending where C_2 begins, we construct a new oriented curve, called $C_1 + C_2$, by joining them together. (See Figure 18.12.) Property 4 is the analogue for line integrals of the property for definite integrals which says that

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx.$$

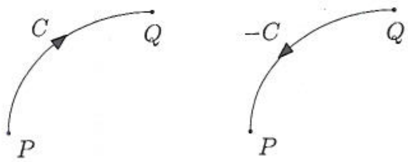


Figure 18.11: A curve, C, and its opposite, $-C$

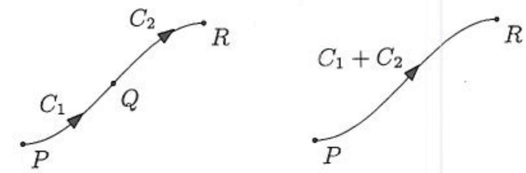


Figure 18.12: Joining two curves, C_1, and C_2, to make a new one, $C_1 + C_2$

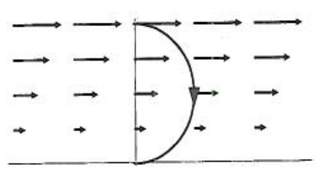
Worksheet #4

Exercises and Problems for Section 18.1

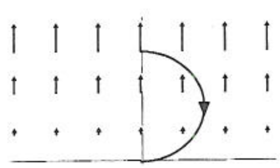
Exercises

In Exercises 1–4, say whether you expect the line integral of the pictured vector field over the given curve to be positive, negative, or zero.

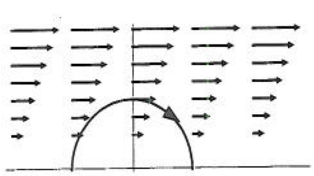
1.



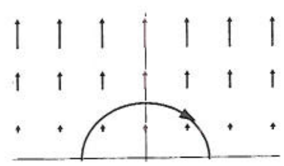
2.



3.



4.



In Exercises 5–10, calculate the line integral of the vector field along the line between the given points.

5. $\vec{F} = x\vec{j}$, from $(1, 0)$ to $(3, 0)$
6. $\vec{F} = x\vec{j}$, from $(2, 0)$ to $(2, 5)$
7. $\vec{F} = x\vec{i}$, from $(2, 0)$ to $(6, 0)$
8. $\vec{F} = x\vec{i} + y\vec{j}$, from $(2, 0)$ to $(6, 0)$
9. $\vec{F} = \vec{r}$, from $(2, 2)$ to $(6, 6)$
10. $\vec{F} = 3\vec{i} + 4\vec{j}$, from $(0, 6)$ to $(0, 13)$

Problems

11. Given the force field $\vec{F}(x, y) = y\vec{i} + x^2\vec{j}$ and the right-angle curve, C, from the points $(0, -1)$ to $(4, -1)$ to $(4, 3)$ shown in Figure 18.13:

    (a) Evaluate $\vec{F}$ at the points $(0, -1)$, $(1, -1)$, $(2, -1)$, $(3, -1)$, $(4, -1)$, $(4, 0)$, $(4, 1)$, $(4, 2)$, $(4, 3)$.
    (b) Make a sketch showing the force field along C.
    (c) Estimate the work done by the indicated force field on an object traversing the curve C.

Figure 18.13

12. (a) For each of the vector fields, $\vec{F}$, shown in Figure 18.14, sketch a curve for which the integral $\int_C \vec{F} \cdot d\vec{r}$ is positive.
(b) For which of the vector fields is it possible to make your answer to part (a) a closed curve?

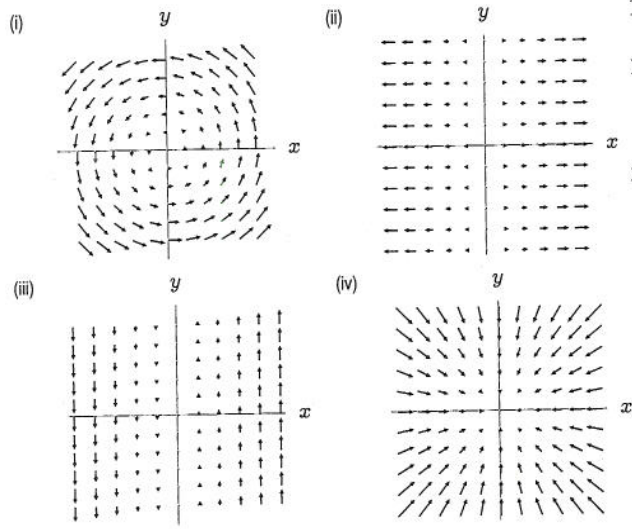


Figure 18.14

13. Consider the vector field $\vec{F}$ shown in Figure 18.15, together with the paths C_1, C_2, and C_3. Arrange the line integrals $\int_{C_1} \vec{F} \cdot d\vec{r}$, $\int_{C_2} \vec{F} \cdot d\vec{r}$ and $\int_{C_3} \vec{F} \cdot d\vec{r}$ in ascending order.

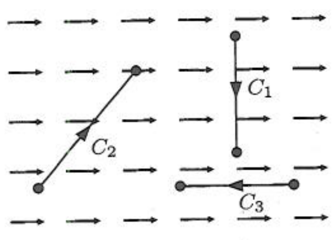


Figure 18.15

For Problems 14–18, say whether you expect the given vector field to have positive, negative, or zero circulation around the closed curve $C = C_1 + C_2 + C_3 + C_4$ in Figure 18.16. The segments C_1 and C_3 are circular arcs centered at the origin; C_2 and C_4 are radial line segments. You may find it helpful to sketch the vector field.

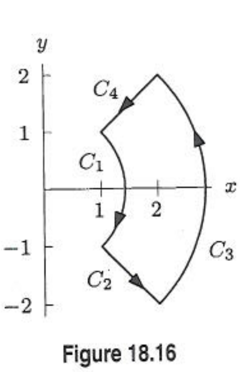


Figure 18.16

14. $\vec{F}(x, y) = x\vec{i} + y\vec{j}$
15. $\vec{F}(x, y) = -y\vec{i} + x\vec{j}$
16. $\vec{F}(x, y) = y\vec{i} - x\vec{j}$
17. $\vec{F}(x, y) = x^2\vec{i}$
18. $\vec{F}(x, y) = -\dfrac{y}{x^2+y^2}\vec{i} + \dfrac{x}{x^2+y^2}\vec{j}$

19. Draw an oriented curve C and a vector field $\vec{F}$ along C that is not always perpendicular to C, but for which $\int_C \vec{F} \cdot d\vec{r} = 0$.

20. Let $\vec{F}$ be the constant force field $\vec{j}$ in Figure 18.17. On which of the paths C_1, C_2, C_3 is zero work done by $\vec{F}$? Explain.

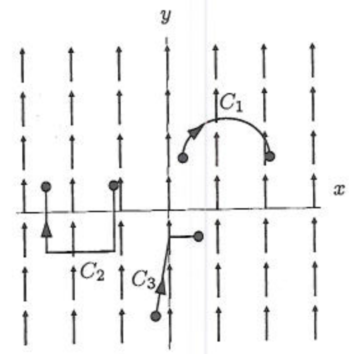


Figure 18.17

21. Explain why the following statement is true: Whenever the line integral of a vector field around every closed curve is zero, the line integral along a curve with fixed endpoints has a constant value independent of the path taken between the endpoints.

22. Explain why the converse to the statement in Problem 21 is also true: Whenever the line integral of a vector field depends only on endpoints and not on paths, the circulation around every closed curve is zero.

23. A square has side 1000 km. A wind blows from the east and decreases in magnitude toward the north at a rate of 6 meter/sec for every 500 km. Compute the circulation of the wind counterclockwise around the square.

In Problems 24–25, use the fact that the force of gravity on a particle of mass m at the point with position vector $\vec{r}$ is

$$\vec{F} = -\frac{GMm\vec{r}}{\|\vec{r}\|^3}$$

where and G is a constant and M is the mass of the earth.

24. Calculate the work done by the force of gravity on a particle of mass m as it moves from 8000 km to 10,000 km from the center of the earth.

25. Calculate the work done by the force of gravity on a particle of mass m as it moves from 8000 km from the center of the earth to infinitely far away.