Worksheet #3 Integration Practice

8. This problem concerns the iterated integral

$$\int_0^1 \int_{y/2}^{(y/2)+2} (2x - y)\, dx\, dy.$$

(a) Evaluate this integral and sketch the region D of integration in the xy-plane.

(b) Let $u = 2x - y$ and $v = y$. Find the region D^* in the uv-plane that corresponds to D.

(c) Use the change of variables theorem (Theorem 5.3) to evaluate the integral by using the substitution $u = 2x - y$, $v = y$.

9. Evaluate the integral

$$\int_0^2 \int_{x/2}^{(x/2)+1} x^5(2y - x)e^{(2y-x)^2}\, dy\, dx$$

by making the substitution $u = x$, $v = 2y - x$.

10. Determine the value of

$$\iint_D \sqrt{\frac{x+y}{x-2y}}\, dA,$$

where D is the region in $\mathbf{R}^2$ enclosed by the lines $y = x/2$, $y = 0$, and $x + y = 1$.

11. Evaluate $\iint_D (2x + y)^2 e^{x-y}\, dA$, where D is the region enclosed by $2x + y = 1$, $2x + y = 4$, $x - y = -1$, and $x - y = 1$.

12. Evaluate

$$\iint_D \frac{(2x + y - 3)^2}{(2y - x + 6)^2}\, dx\, dy,$$

where D is the square with vertices $(0, 0)$, $(2, 1)$, $(3, -1)$, and $(1, -2)$. (Hint: First sketch D and find the equations of its sides.)

In Exercises 13–17, transform the given integral in Cartesian coordinates to one in polar coordinates and evaluate the polar integral.

13. $\displaystyle\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 3\, dy\, dx$

14. $\displaystyle\int_0^2 \int_0^{\sqrt{4-x^2}} dy\, dx$

15. $\displaystyle\iint_D (x^2 + y^2)^{3/2}\, dA$, where D is the disk $x^2 + y^2 \le 9$

16. $\displaystyle\int_{-a}^{a} \int_0^{\sqrt{a^2-y^2}} e^{x^2+y^2}\, dx\, dy$

17. $\displaystyle\int_0^3 \int_0^x \frac{dy\, dx}{\sqrt{x^2 + y^2}}$

18. Evaluate

$$\iint_D \frac{1}{\sqrt{4 - x^2 - y^2}}\, dA,$$

where D is the disk of radius 1 with center at $(0, 1)$. (Be careful when you describe D.)

19. Let D be the region between the square with vertices $(1, 1)$, $(-1, 1)$, $(-1, -1)$, $(1, -1)$ and the unit disc centered at the origin. Evaluate $\displaystyle\iint_D y^2\, dA$.

20. Find the total area enclosed inside the rose $r = \sin 2$ (Hint: Sketch the curve and find the area inside a single leaf.)

21. Find the area of the region inside the cardioid $r = 1 - \cos\theta$ and outside the circle $r = 1$.

22. Find the area of the region bounded by the positive x-axis and the spiral $r = 3\theta$, $0 \le \theta \le 2\pi$.

23. Evaluate

$$\iint_D \cos(x^2 + y^2)\, dA,$$

where D is the shaded region in Figure 5.100.

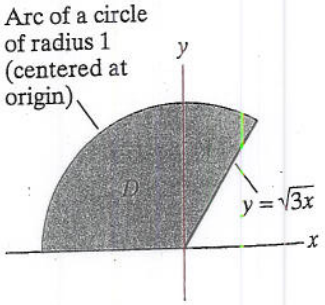

Figure 5.100 The region D of Exercise 23.

24. Evaluate

$$\iiint_B \frac{dV}{\sqrt{x^2 + y^2 + z^2 + 3}},$$

where B is the ball of radius 2 centered at

Text: Review problems (pp 417-420)
#1, 2, 3, 23 (as described in class), 33