Math 550 Worksheet 2
Spring, 2008

You are free, in fact encouraged, to collaborate, and to bring questions about these to class.

1. Let $A = \begin{bmatrix} 3 & 0 & -2 \\ 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ and $T_A \colon \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by left multiplication by A on column vecors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$. Compute the vectors $\mathbf{a} = T_A(\hat{\mathbf{i}})$, $\mathbf{b} = T_A(\hat{\mathbf{j}})$, and $\mathbf{c} = T_A(\hat{\mathbf{k}})$. Compute, by any method, the volume of the parallelepiped D spanned by the vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$. The unit cube $D^* = \{(x, y, z) \mid 0 \le x, y, z \le 1\}$ or $[0,1] \times [0,1] \times [0,1]$ is carried one-to-one and onto D by T_A. Compute all eight vertices of D and sketch it if you can (hint: the vertices of D^* are $(0,0,0)$ and the tips of vectors such as $\hat{\mathbf{i}} + \hat{\mathbf{k}}$, $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, etc.).

2. Rewrite the integral $\int\int_D \sqrt{\dfrac{x+y}{x-2y}}\, dA$ as an integral over a region D^* in the uv-plane, where D is the region enclosed by $y = x/2$, the x-axis, and $x + y = 1$. The new integral should be completely set up, so that it is clear how one would compute it; but do NOT compute it. Do, however, inspect your answer and see if you find anything troubling about it. Does this trouble actually appeasr in the original problem? Hint: make a diagram, decide on substitutions $u =\ $, $v =\ $, describe the region D^* in the uv-plane so that $(x, y) = T(u, v)$ and $D = T(D^*)$, and use the change of variables theorem. You don't actually need to find a formula for T at this point because of the following hint within the hint: you may find it useful to know that $\dfrac{\partial(x,y)}{\partial(u,v)}$ is actually the reciprocal of $\dfrac{\partial(u,v)}{\partial(x,y)}$. This in turn can be verified by solving for (x, y) in terms of (u, v), which amounts to finding a formula for T, and computing $\dfrac{\partial(x,y)}{\partial(u,v)}$ directly.

3. Compute $\dfrac{\partial(x,y,z)}{\partial(\rho,\varphi,\theta)}$. Explain why this determinant is already positive, when using conventional spherical coordinates.