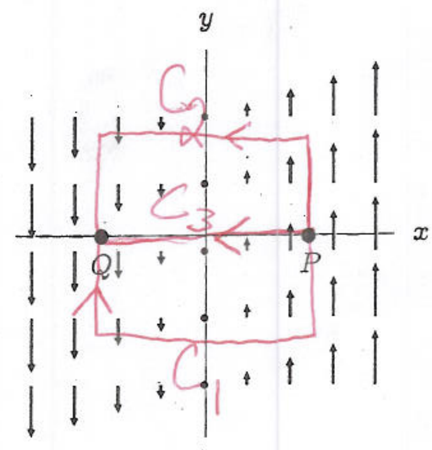
MATH 550 **Spring, 2006** **Quiz #4** **Name:** M^2

1. A vector field $\mathbf{F}$ is illustrated below. Find a path C_1 from P to Q so that $\int_{C_1} \mathbf{F} \cdot d\mathbf{s}$ is negative, a path C_2 from P to Q so that $\int_{C_2} \mathbf{F} \cdot d\mathbf{s}$ is positive, and a path C_3 from P to Q so that $\int_{C_3} \mathbf{F} \cdot d\mathbf{s}$ is zero. What conclusion can you draw concerning $\mathbf{F}$?

$\vec{F}$ can not be a gradient vector field (is not "conservative") since independence of path fails.



2. Compute dS (scalar) for the surface given by $x = u^3$, $y = 1/v$, $z = e^{-uv}$.

$$(x, y, z) = \vec{T}(u, v) = \left(u^3, \frac{1}{v}, e^{-uv}\right)$$

$$\vec{T}_u = (3u^2, 0, -ve^{-uv})$$

$$\vec{T}_v = \left(0, -\frac{1}{v^2}, -ue^{-uv}\right)$$

$$\vec{T}_u \times \vec{T}_v = \left(\frac{1}{v} e^{-uv}, 3u^3 e^{-uv}, -\frac{3u^2}{v^2}\right)$$

$$dS = \| \vec{T}_u \times \vec{T}_v \| \, du\,dv$$

$$= \left(\frac{1}{v^2} e^{-2uv} + 9u^6 e^{-2uv} + \frac{9u^4}{v^4}\right)^{\frac{1}{2}} du\,dv$$

(over →)

3. Compute $\oint_C \overbrace{(yz^2 - y, xz^2 + x, 2xyz)}^{\vec{F}} \cdot d\mathbf{s}$, where C is the circle of radius 3 centered at the $(0, 0, 1)$ in the plane $z = 1$, starting and ending at $(3, 0, 1)$, oriented counterclockwise (there is a well-known parameterization of C using the polar coordinate θ, which you may rename t if you like, so that you have $\mathbf{c}(t) = (x(t), y(t), z(t))$. Recall that $\int \sin^2 t\, dt = (2t - \sin(2t))/4$ and $\int \cos^2 t\, dt = (2t + \sin(2t))/4$.

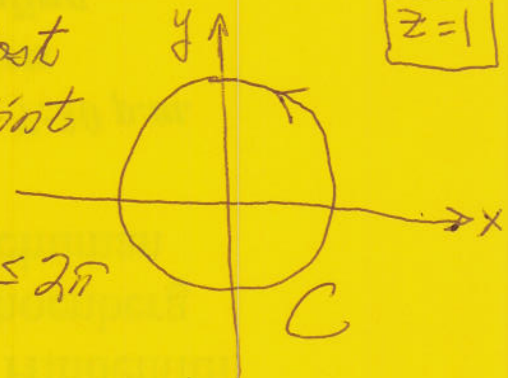
$\boxed{z=1}$

$x = 3\cos t$

$y = 3\sin t$

$z = 1$

$0 \le t \le 2\pi$



$$d\vec{s} = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right)dt$$

$$= (-3\sin t, 3\cos t, 0)\,dt$$

Along C, $z = 1$,

and $(yz^2 - y, xz^2 + x, 2xyz)$

$$= (0, 2x, 2xy)$$

$$= (0, 6\cos t, (6\cos t)(3\sin t))$$

So $\oint_C \vec{F} \cdot d\vec{s} = \int_0^{2\pi} 18\cos^2 t\, dt = \frac{18}{4}(2t + \sin 2t)\Big|_0^{2\pi}$

$$= 18\pi$$