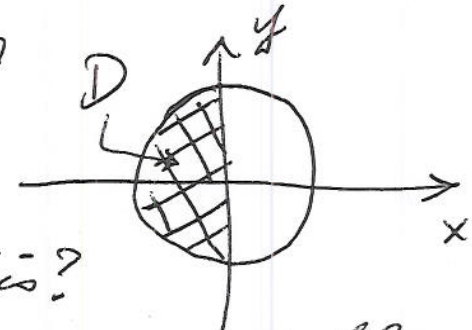
MATH 550 **Spring, 2008** **Quiz #3** **Name:** M^2

1. Let D be the left half of the unit disc. Rewrite $\iint_D (1+x^2+y^2)^{3/2}\,dx\,dy$ completely in polar coordinates. It is not necessary to do the integration.

$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\int_0^1 (1+r^2)^{3/2}\, r\,dr\,d\theta$$



(The value turns out to be $\frac{4\pi}{5}\sqrt{2}$. Can you get this? Why is this change of variables so advantageous?)

2. a. Compute $\iiint_D \frac{dx\,dy\,dz}{(x^2+y^2+z^2)^{5/4}}$, where D is the region outside the sphere $x^2+y^2+z^2=\delta^2$ and inside $x^2+y^2+z^2=81$.

   b. Use the result of part (a) to compute the integral $\iiint_B \frac{dx\,dy\,dz}{(x^2+y^2+z^2)^{5/4}}$, where B is the solid ball of radius 9, centered at the origin. Why is it necessary to use part (a)?

(a) Change to spherical coordinates:

$$\int_0^{2\pi}\int_0^{\pi}\int_\delta^9 \frac{\rho^2 \sin\varphi\,d\rho\,d\varphi\,d\theta}{(\rho^2)^{5/4}} \qquad \frac{\rho^{4/2}}{\rho^{5/2}} = \frac{1}{\rho^{\frac{1}{2}}}$$

$$= \int_0^{2\pi}\int_0^{\pi}\int_\delta^9 \rho^{-\frac{1}{2}} \sin\varphi\,d\rho\,d\varphi\,d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi}\left(2\rho^{\frac{1}{2}}\Big|_\delta^9\right)\sin\varphi\,d\varphi\,d\theta = 2(3-\sqrt{\delta})\int_0^{2\pi}\left(-\cos\varphi\Big|_0^{\pi}\right)d\theta$$

$$= 2(3-\sqrt{\delta})\underbrace{(-(-1)+1)}_{2}\int_0^{2\pi} d\theta = 8\pi(3-\sqrt{\delta})$$

(b) The integral over B is improper because of division by 0 at $(0,0,0)$. But if we take $\lim_{\delta\to 0^+}$ of the result of (a), we obtain

$$\iiint_B \frac{dx\,dy\,dz}{(x^2+y^2+z^2)^{5/4}} = \lim_{\delta\to 0^+} 8\pi(3-\sqrt{\delta}) = 24\pi.$$