MATH 550 **Spring, 2008** **Quiz #1** **Name:** M^2

1. The vector field $\mathbf{F} = (2xy^3, 3x^2y^2 + z\cos(yz), y\cos(yz))$ is in fact a gradient vector field in $\mathbb{R}^3$. Compute the most general scalar field $f(x,y,z)$ for which $\mathbf{F} = \operatorname{grad} f = \nabla f$. If in addition one knows that $f(1,0,1) = 3$, find the exact formula for f.

Solve $\frac{\partial f}{\partial x} = 2xy^3$, $\frac{\partial f}{\partial y} = 3x^2y^2 + \cos(yz)$,

$\frac{\partial f}{\partial z} = y\cos(y,z)$ for $f(x,y,z)$ by anti-differentiating (integrating).

$f = x^2y^3 + A(y,z)$

$f = x^2y^3 + \sin(yz) + B(x,z)$

$f = \sin(yz) + C(x,y)$.

ordinary constant

Compare: $f(x,y,z) = x^2y^3 + \sin(yz) + C$

$3 = f(1,0,1) = 0 + 0 + C$ so $C = 3$

$f(x,y,z) = x^2y^3 + \sin(yz) + 3$

2. Let $A = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & -1 \\ 2 & 1 & 0 \end{bmatrix}$ and $T_A \colon \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by left multiplication by A on column vecors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$. Compute the vectors $\mathbf{a} = T_A(\hat{\mathbf{i}})$, $\mathbf{b} = T_A(\hat{\mathbf{j}})$, and $\mathbf{c} = T_A(\hat{\mathbf{k}})$. Compute, by any method, the volume of the parallelepiped spanned by the vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$.

$\vec{a} = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & -1 \\ 2 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} = A\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\vec{c} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} = A\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

T_A carries the unit cube $[0,1]\times[0,1]\times[0,1]$ to the parallelepiped spanned by $\vec{a}$, $\vec{b}$, and $\vec{c}$. The cube has vol $= 1$, and the multiplier is $|\det A| = |1(1) - (-2)(2)| = |5| = 5$.