MATH 550 Vector Analysis
Project Ideas for Spring, 2008

The basics. You may do the project alone or in groups of at most two people. The choice of a theme is up to you, but must have my approval before you proceed. My interest will be that it involves some serious mathematics related to the course content, and that you will have to give some sort of clean, well-prepared account of what you have done, generally in writing, but perhaps also in a compressed form as a class presentation. Some projects will consist of linked sequences of non-trivial exercises from the text or elsewhere, others will involve readings in other books or in properly vetted websites. In any event you must DO some mathematics, not just tell a story about it. Plagiarism of student projects or faculty demos (like animations of ants crawling on Moebius strips) that have been posted on the web or otherwise obtained is strictly forbidden; don't waste your time plagiarizing Wikipedia–I know about it. In general I will be very suspicious of web-based material; use the web to find good print sources, then use those. Be prepared to answer questions, either in class or in my office; you can expect that I will be able to detect work that is not your own. Keep in mind that the whole project is only worth 35 points so don't invest your life in it (unless you want to); if some of these projects turn out to be overwhelming, but you have already invested a lot of time, let me know and we can adjust. You are welcome, indeed invited, to discuss your progress as you go.

You may want to present this list to other prof's in physics, geology, chemistry, or engineering, and ask their opinion which project would be "good for you." Presentations of about 15-20 minutes will be welcome the last week (and may contribute pints to your score). The final deadline is April 24.

Possibilities.

(1) Do exercises 12-17 of section 4.2 on pages 282–284. For extra credit compute the total curvature of the curve starting with the helix for $0 \le t \le 2\pi$, and closing up by continuing to corkscrew down for $2\pi \le t \le 4\pi$. This won't be quite as long and hard as it may seem because a large part of the solution is in the back of the book. I would expect you to be able to give a short presentation or to answer questions.

(2) Do problem 20 from section 7.2. Are the processes of (b), (c), and (d) adiabatic? In addition, show me a similar problem from a physical chemistry text that involves a gas that is not "van der Waals", and work such a problem.

(3) The laws of Faraday and Ampère are discussed in about half a dozen places in the text. Do the indicated exercises and prepare a short presentation on the highlights of the theory to the class.

(4) Do a complete and careful job of exercises 8-11 from section 6.1. Do not use high powered theorems from linear algebra, but get your hands dirty with the 2×2 matrices involved. Do the analogous problems for linear transformations of $\mathbb{R}^3$; you may use results from linear algebra, but try to

minimize this. Prepare a short presentation of the highlights to the class.

(5) Soap films, minimal surfaces, and Plateau's problem. This project literally calls for drippy "hand-waving", but I expect some serious mathematical content as well (for example, some of the exercises of section 7.7). There has been some very exciting and surprising progress recently (2003) on the infinite case (look for an article on Minimal Surfaces in the *Notices* of the American Mathematical Society (AMS)).

(6) Discuss curvature for curves and/or surfaces. The definitions in terms of local coordinate computations are tedious but straightforward; what is more interesting is "global" or "total" or "average" curvature. There are a number of definitions, of which the text touches on some, and has exercises on them. There would be a number of things you could do. One would be to get a parameterization of the trefoil knot, and to compute the total curvature of it and of a circle of radius R, and to compare these (see pp. 425–427). Another would be to investigate the quantities E, G, F, H, and K of exercise 15 on p. 481 and section 7.7; and then to give an account of the Gauss-Bonnet Theorem and the meaning of "genus"; I'd expect you to do some of the exercises of 7.7.

(7) Do exercises 21-26 on pages 530–531. As many parts of the solution are given, I would expect that you could respond to questions on this material. From the text (or elsewhere), give some examples in which Laplace's equation is used, even better solve an explicit problem. One possibility would be to describe the problem of "hearing the shape of a drum", when you can and when you cannot. Most of this will be too technical to read, never mind to write up and present, but a rough outline of what mathematicians did with this interesting problem should be possible. In this case the Wikipedia article is a good place to start, and even Google brings up some good sources.

(8) Not mentioned in the text, but maybe interesting if you are wondering about inward and outward pointing normals, is the "sphere eversion" problem, that is, how to turn a sphere inside out, by allowing self crossings, but no singularities such as points or ridges (if you try the obvious thing and push the south pole up through the north pole you will pass through a roundish configuration with a ridge along the equator). I believe the existence of such a motion was abstractly proved in the '60's, probably by Smale, later done explicitly by a (blind) French mathematician Morin. George Francis of the University of Illinois was one of the first to draw some actual pictures (originally with colored chalk on a blackboard!), and now animations are a dime a dozen on the web. The main problem would be to extract some real information from all this.

(9) Feel free to suggest other topics, and I will also keep my eyes open to further possibilities.