MATH 550 **Bonus problem** **Name:**_____________________

Spring, 2006

As I mentioned in class, any exchange of limits is problematic, and the exchange of a partial derivative with an integral (both of which are limits) is no exception. This problem, worth up to 10 points, illustrates the difficulty in exchanging a limit with an integral. It is due on April 13.

1. Let $g(y,t)$ be defined as follows for $0 < y \leq 1/2$ and $0 \leq t \leq 1$. Suggestion: make a sketch of $z = g(y,t)$ for a selection of fixed values of y, giving z as a function of t.

$$g(y,t) = \begin{cases} \frac{2}{y^2}t & \text{for } 0 \leq t \leq y \\ \frac{4}{y} - \frac{2}{y^2}t & \text{for } y \leq t \leq 2y \\ 0 & \text{for } 2y \leq t \leq 1 \end{cases}$$

a. Compute, for fixed t, $\lim_{y\to 0} g(y,t)$. Compute $\int_0^1 \lim_{y\to 0} g(y,t)\, dt$.

b. Compute, for fixed y, $\int_0^1 g(y,t)\, dt$, and then compute $\lim_{y\to 0} \int_0^1 g(y,t)\, dt$.

c. Give an intuitive explanation for the discrepancy between these two limits. What sort of hypothesis on $g(y,t)$ might prevent a discrepancy like this one?