

3. (16 points) Rewrite the integral $\iint_D (x+y)^2 e^{-3x-y} dy dx$ as a simpler integral over a region D^* in the uv -plane, where D is the region enclosed by $y = -3x$, the x -axis, and $x+y = 2$. Note: the transformation T will be linear so corners go to corners and edges go to edges. The new integral should be completely set up, so that it is clear how one would compute it; but do NOT compute it.

4. (8 points) Compute dS (scalar) for the surface given by $x = s^3$, $y = 1/t$, $z = e^{-st}$.

5. (16 points) Let D be the solid region bounded by $x^2 + z^2 = 9$, $y = -1$, and $y = 4$, let $S = \partial D$, and let $\hat{\mathbf{n}}$ be the unit outward pointing normal. Compute $\iint_S \mathbf{F} \cdot \hat{\mathbf{n}} dS$ for $\mathbf{F} = y^3 \hat{\mathbf{j}}$.

6. (10 points) Compute $\int_C \mathbf{F} \cdot d\mathbf{s}$ for $\mathbf{F} = x \hat{\mathbf{j}} + z \hat{\mathbf{k}}$, C the line segment from $(1, 3, 2)$ to $(4, -1, 2)$.

7. (20 points) a. Without computing a potential function for the vector field $\mathbf{F} = (2x \sin y, x^2 \cos y, 1/(z + 1))$, explain why \mathbf{F} must be conservative in the “upper half-space” where $z > -1$.

b. Now compute a potential function for \mathbf{F} in this half space.

c. Suppose C is any path from $P(0, 0, 0)$ to $Q(1, \pi/2, 0)$ that lies in this half space. Compute $\int_C \mathbf{F} \cdot ds$.

8. (12 points) Compute $\int_C \mathbf{F} \cdot d\mathbf{s}$ for $\mathbf{F} = (6xz - 2y)\hat{\mathbf{i}} + (z^3 - 2x)\hat{\mathbf{j}} + (3x^2 + 3z^2y)\hat{\mathbf{k}}$, C the circle $(x - 2)^2 + (y - 1)^2 = 1$ oriented counterclockwise in the plane $z = 1$. Hint: C is the boundary of a nice surface S .

9. (8 points) A vector field \mathbf{F} is illustrated below, with a path C made up of C_1 , C_2 , C_3 and C_4 in this order.

a. Determine if $\int_C \mathbf{F} \cdot d\mathbf{s}$ is negative, positive, or zero.

b. Does \mathbf{F} have the “independence of path” property in the region shown? Explain, briefly.