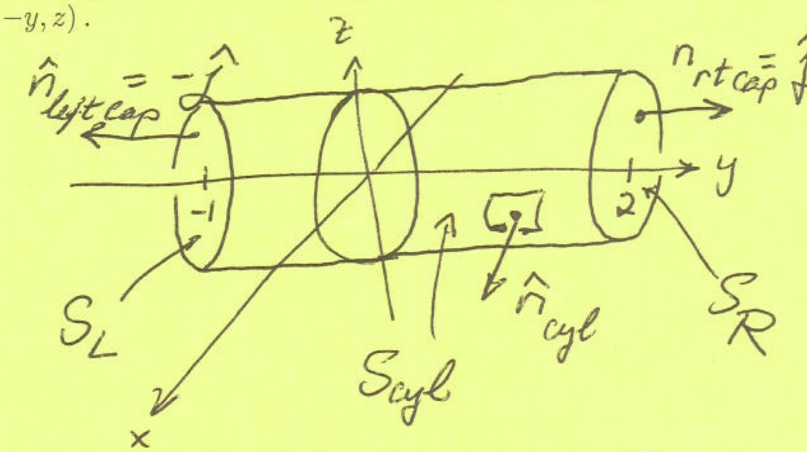
7. (26 points) Compute the flux of $\mathbf{F}$, *i.e.*, $\iint_S \mathbf{F}\cdot d\mathbf{S}$, through the closed cylinder S given by $y=-1$, $x^2+z^2=25$, $y=2$ and oriented by outward pointing normal vectors, where $\mathbf{F}=(x,-y,z)$.

Compute flux through the ends first.



Left $\displaystyle\iint_{S_L} \vec{F}\cdot d\vec{S}$

$$= \iint_{S_L} (x,-(-1),z)\cdot(0,-1,0)\,dS = -\iint_{S_L} dS$$

$$= -\text{Area of } S_1 = -25\pi$$

Right $\displaystyle\iint_{S_R} \vec{F}\cdot d\vec{S} = \iint_{S_R} (x,-2,z)\cdot(0,1,0)\,dS$

$$= -2\iint_{S_R} dS = -2\,\text{Area}(S_R) = -50\pi$$

Now the sides:

S_{cyl} is parameterized by $\left.\begin{array}{l} x = 5\cos t \\ y = y \\ z = 5\sin t \end{array}\right\}$ $\begin{array}{l} 0\le t\le 2\pi \\ -1\le y\le 2 \end{array}$

$\vec{T}_t = (-5\sin t, 0, 5\cos t)$

$\vec{T}_y = (0,1,0)$

$\vec{T}_t \times \vec{T}_y = (-5\cos t, 0, -5\sin t)$

Notice at $t=0, y=0$, this is pointing inward, so we better reverse it.

Use $d\vec{S} = (5\cos t, 0, 5\sin t)\,dy\,dt$

$\vec{F} = (5\cos t, -y, 5\sin t)$, so $\vec{F}\cdot d\vec{S} = (25\cos^2 t + 25\sin^2 t)\,dy\,dt = 25\,dy\,dt$

$$\iint_{S_{cyl}} \vec{F}\cdot d\vec{S} = \int_0^{2\pi}\int_{-1}^{2} 25\,dy\,dt = 150\pi$$

$$\iint_S \vec{F}\cdot d\vec{S} = 75\pi$$