MATH 550 Spring, 2006 Exam #2 Name:____________________

General instructions. Show your work. If you use major results quote them by name. Compute integrals in the easiest way that you can find; if your computation is turning out to be very complicated, think if a different approach might be possible. There are 120 points.

Stokes' Theorem. Let S be a bounded, piecewise regular, oriented surface in $\mathbb{R}^3$ and suppose that $C = \partial S$ consists of finitely many piecewise smooth simple closed curves, oriented consistently with the orientation of S. Suppose that $\mathbf{F}$ is a C^1 vector field defined on a domain that includes S. Then $\iint_S \vec{\nabla} \times \mathbf{F} \cdot d\mathbf{S} = \iint_S \vec{\nabla} \times \mathbf{F} \cdot \hat{\mathbf{n}}\, dS = \oint_C \mathbf{F} \cdot d\mathbf{s}$. Recall that Green's Theorem is a special case that applies when S lies in the plane and $\hat{\mathbf{n}} = \hat{\mathbf{k}}$.

1. (9 points) Assume $\mathbf{F}$ is a C^1 vector field in all of $\mathbb{R}^3$. Give three properties of $\mathbf{F}$ if it happens to have the form $\vec{\nabla} f$.

2. (12 points) a. Show how to subdivide the following region D in the plane so that Green's Theorem can be applied. Show how to orient ∂D so that the orienting normal vector is $-\hat{\mathbf{k}}$.

   b. For the vector field $\mathbf{F}$ shown below, give oriented paths C_1, C_2, and C_3 so that $\int_{C_i} \mathbf{F} \cdot d\mathbf{s}$ is positive, zero, and negative respectively.

3. (12 points) Let $\Phi\colon D \to S$ be given by $(x, y, z) = \Phi(u, v) = (uv^2, u^3, v)$ on $D = [0,1] \times [0,1]$ (the unit square).

a. Compute $d\mathbf{S}$ (vector) so that it (or $\hat{\mathbf{n}}$) points upward.

b. Compute dS (scalar).

4. (20 points) Let $\mathbf{F} = (0, y/(y^2 + z^2), z/(y^2 + z^2))$ and C be given by $x = 0$, $y = 3\cos t$, $z = 2\sin t$, $0 \le t \le \pi/2$. Compute the work done by $\mathbf{F}$, *i.e.*, $\int_C \mathbf{F} \cdot d\mathbf{s}$.

5. (26 points) Compute $\iint_S \vec{\nabla} \times \mathbf{F} \cdot d\mathbf{S}$ for $\mathbf{F} = (3z^2 + yz, -y^2, x)$, and S the portion of the surface $z = 10 - (x^2 + y^2)$ that lies above the plane $z = 1$ and is oriented so that $\hat{\mathbf{n}}$ points upward and away from the origin.

6. (15 points) Let $\mathbf{F} = (z, 0, -x)$ and C be the line segment from $(1, 0, 2)$ to $(3, 1, 1)$. Compute $\int_C \mathbf{F} \cdot d\mathbf{s}$.

7. (26 points) Compute the flux of $\mathbf{F}$, *i.e.*, $\iint_S \mathbf{F} \cdot d\mathbf{S}$, through the closed cylinder S given by $y = -1$, $x^2 + z^2 = 25$, $y = 2$ and oriented by outward pointing normal vectors, where $\mathbf{F} = (x, -y, z)$.