MATH 550 **Spring, 2006** **Exam #1** **Name:**______________________

Note! For full credit you must show sufficient work to support your answer. There are 120 points. *Good luck!*

Change of Variables Theorem. In two variables, $\iint_D f(x,y)\,dx\,dy = \iint_{D^*} f(T(u,v))|\frac{\partial(x,y)}{\partial(u,v)}|\,du\,dv$ and in three variables $\iiint_D f(x,y,z)\,dx\,dy\,dz = \iiint_{D^*} f(T(u,v,w))|\frac{\partial(x,y,z)}{\partial(u,v,w)}|\,du\,dv\,dw$, where D and D^* are suitable regions and T is a suitable transformation such that $T(D^*) = D$.

1. (5 points) Give dV in spherical coordinates.

2. (15 points) Let $x = v + \ln(1+w^2)$, $y = \cos u$, and $z = vw$. Compute $\dfrac{\partial(x,y,z)}{\partial(u,v,w)}$. If $(x,y,z) = T(u,v,w)$ is given by the formulas above, what is the volume change factor in the vicinity of the point $(u,v,w) = (\pi/2, -1, -1)$?

3. (12 points) Explain why $\mathbf{F} = (F_1, F_2) = (\sin x - x\cos x + e^{xy^2}, 2e^{xy^2})$ can NOT be a gradient vector field $\vec{\nabla} f$.

4. (17 points) Let $\mathbf{F} = (xe^{-x^2} + y\cos(xy), x\cos(xy) + 3y^2)$. This is a gradient vecor field; find the scalar function $f(x,y)$ so that $\mathbf{F} = \vec{\nabla} f$.

5. (17 points) Let C be the curve given by $\mathbf{c}(t) = (\sin(3t), \cos(3t))$ on the interval $0 \leq t \leq \pi/6$. Compute $\int_C xy\, ds$.

6. (17 points) Find a linear transformation $(x, y) = T(u, v) = (au + bv, cu + dv)$ that carries the unit square $D^* = [1, 0] \times [0, 1]$ to the parallelogram D with corners at $(0, 0)$, $(-2, 1)$, $(-1, 2)$, and $(1, 1)$. What is the area of D? Does T preserve or reverse orientation, and how do you know?

7. (17 points) Compute $\displaystyle\iint_A \frac{1}{(x^2 + y^2)^{7/8}}\, dx\, dy$ over the annulus (ring) A between the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$, where $0 < a < b$. Use this result to compute the same integral over the disc $D = \{(x, y) \mid x^2 + y^2 \leq 16\}$, and explain why the first result is really needed.

8. (20 points) Consider the region D in the first quadrant bounded by the curves $xy = 1$, $xy = 3$, $x^2 - y^2 = 1$, $x^2 - y^2 = 4$, as illustrated below. Compute the integral $\iint_D (x^2 + y^2)\, dx\, dy$ by making a suitable change of variables. You may (although this is not strictly necessary with a little bit of insight) want to express $x^2 + y^2$ in terms of $u^2 + 4v^2$ or $v^2 + 4u^2$ (depending on what you call u and what you call v). It is difficult, and not helpful, to solve for x and y individually in terms of u and v, and it is best to compute $\frac{\partial(x,y)}{\partial(u,v)}$ indirectly.