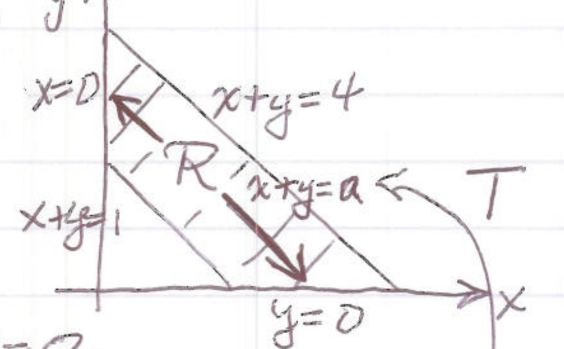
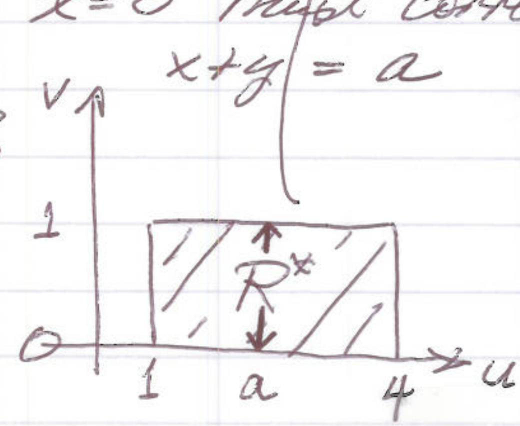
6.2 #8 $\displaystyle\iint_R \frac{1}{x+y}\,dy\,dx$ R bounded by

$x=0$, $y=0$, $x+y=1$, $x+y=4$. Use $T(u,v) = (u-uv,\ uv) = (x,y)$.



Notice that $y=0$ corresponds to $uv=0$, so $u=0$ or $v=0$.
If $u=0$ then also $x=0$, but $(0,0)$ is not in R. Also observe that $\boxed{x+y = u-uv+uv = u}$ so $x+y=1$ corres. to $u=1$, $x+y=4$ to $u=4$.
Finally, $x=0$ corres. to $u-uv=0$ or $u(1-v)=0$. If $u=0$ then $uv=y=0$, and again this is not possible since $(0,0) \notin R$. So $x=0$ must corres. to $v=1$. The lines $x+y=a$ with $1 \le a \le 4$ fill R, and correspond to vertical segments $u=a$ in R^*.



$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 1-v & -u \\ v & u \end{vmatrix} = u - vu + uv = u$$

$$\iint_R \frac{1}{x+y}\,dy\,dx = \int_1^4\int_0^1 \frac{1}{u}\,u\,dv\,du = 3.$$

6.2 #12 Any of these substitutions will work: $\begin{cases} x=u \\ y=uv \end{cases}$ $\begin{cases} x=u \\ y=v^2 \\ \text{or } v=\sqrt{y} \end{cases}$ $\begin{cases} x=\sqrt{u}\ (\text{or } u=x^2) \\ y=v \end{cases}$